Financial Intermediation in Economies with Investment Complementarities

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Tese de Doutoramento em Economia

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Biographical Note

Joana was born on January 22,1982, in Viana do Castelo, Portugal. She concluded her graduate studies in Applied Mathematics in 2005, at the Faculdade de Ciências da Universidade do Porto. In the same year, she initiated her professional activity at Banco Comercial Português as risk analyst, where she keeps working.

In 2009 she decided to start the PhD in Economics at the Faculdade de Economia da Universidade do Porto, under the supervision of Professor José Jorge. Her research has been presented in scientific meetings and international conferences, as the 6th International Finance and Banking Society 2014 Conference, the XXIII International Rome Conference on Money, Banking and Finance, the 2015 Financial Management Association European Conference, as well as, WIP Seminars at the Faculdade de Economia da Universidade do Porto.

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Resumo

A tese é composta por três ensaios sobre complementaridades estratégicas e intermediação financeira, incluindo trabalho teórico e empírico. O primeiro ensaio é um estudo sobre *global games* aplicados à macroeconomia e às finanças. O segundo ensaio mostra como a intermediação financeira atenua a falta de coordenação dos indivíduos com informação privada, quando o retorno individual é crescente no investimento agregado. O terceiro ensaio destaca o papel das complementaridades estratégicas na relação entre o crédito bancário e o investimento.

O primeiro ensaio resolve *global games* aplicados à macroeconomia e às finanças. Nele analisamos o papel da informação pública e privada na determinação do equilíbrio único e discutimos o papel dos preços de mercado. Examinamos também o impacto da informação pública no bem-estar social, comparando modelos com e sem complementaridades a nível agregado. Reunimos num único ensaio as contribuições mais relevantes no tópico.

Indivíduos descentralizados não internalizam as externalidades positivas do seu investimento no retorno dos outros, investindo assim muito pouco. Como é que os intermediários financeiros permitem uma coordenação mais eficaz no mercado? Como é que os bancos afetam o investimento e o bem-estar social em equilíbrio? A nossa análise baseia-se no modelo de Angeletos e Pavan (2004), adicionando ao seu modelo intermediários financeiros (também chamados de bancos); os bancos são capazes de reduzir a volatilidade dos fundamentos económicos subjacentes às empresas, monitorizando-as e, como resultado, os bancos são capazes de oferecer produtos financeiros que pagam um retorno relativamente constante.

Analisando as indústrias Portugueses no período entre 2006 e 2012, testamos o nosso modelo teórico no terceiro ensaio, mostrando que as indústrias com fortes complementaridades estratégicas revelam maior sensibilidade à contração de crédito. Mais especificamente, a contração de crédito deve ter impacto diferente entre indústrias e áreas geográficas. Conduzimos o nosso estudo utilizando a base de dados Sabi.

Abstract

The thesis comprises three essays on strategic complementarities and banking, including theoretical and empirical work. The first essay is a primer on global games applied to macroeconomics and finance. The second essay shows how financial intermediation mitigates the coordination failure for individuals with private information, when returns are increasing in the aggregate level of investment. The third essay sheds light on the role of strategic complementarities in the relationship between bank lending and investment.

The first essay shows how to solve global games applied to macroeconomics and finance. We ascertain the roles of public and private information for the determination of a unique equilibrium, and discuss the informative role of market prices. We also examine the impact of public information on social welfare, comparing models with and without complementarities at the aggregate level. We collected in a single paper the most relevant contributions in the field.

Decentralized individuals do not internalize the positive externality of their investment on the return of others, thereby investing too little. How do financial intermediaries permit more effective coordination in the market? How do banks affect equilibrium allocations and social welfare? Our analysis builds on the model of Angeletos and Pavan (2004) and we extend their model by adding financial intermediaries (also called banks); banks are able to reduce the volatility of the underlying economic fundamentals of individual firms by monitoring them and, as a result, banks are able to offer financial products which pay a relatively constant return.

By analyzing Portuguese operating firms for the period between 2006 and 2012, we test our theoretical model in the third essay, showing that industries with strong strategic complementarities exhibit stronger sensitivity to a credit contraction. More specifically, a credit contraction should have different impact across industries and geographical areas. We conduct our study using Sabi database.

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A Primer on Global Games Applied to Macroeconomics and Finance

Essay 1

June 2014

Abstract

This paper shows how to solve global games applied to macroeconomics and finance. We ascertain the roles of public and private information for the determination of a unique equilibrium, and discuss the informative role of market prices. We examine the impact of public information on social welfare, comparing models with and without complementarities at the aggregate level.

Keywords: Global Games, Financial Crises

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1 Introduction

Global games are static coordination games, with strategic complementarities and incomplete information.[1] In this class of games, a player's playoff depends on his own action, the actions of other players, and some unknown economic fundamental. Because of incomplete information, individual actions are motivated not only by beliefs about the unknown economic fundamental, but also by beliefs about beliefs held by other players.

Carlsson and van Damme (1993) show that a global game in which private information is sufficiently precise has a unique equilibrium. This class of games has been extensively applied to macroeconomics and finance, since the contribution by Morris and Shin (1998). Our paper characterizes and relates recent applications in the literature, classifies a variety of models according to strategies and payoffs, and clarifies the role of the main assumptions within each contribution.

Comparing results across models is hard, because of the diversity of setups applied in the literature. To overcome this difficulty, we chose to rewrite the main models with homogeneous notation. We use u to denote the individual payoff functions and W for social welfare, we use k to denote individual investment and a to denote individual actions, we use θ for the fundamental, z for the public signal and x for the private signal, and we use α and β for the precision of public and private information. Finally, we use λ to parameterize the degree of complementarity.

We identify the principles that guide the resolution of each model, and we pinpoint the underlying assumptions responsible for debates in the literature. We start with the contribution of Morris and Shin (2000), which illustrates how adding a small amount of (private) noise to a game with multiple equilibria removes the multiplicity.[2] With a unique equilibrium, it is possible to perform safe comparative-statics analysis which, in turn, assists policy analysis. We present the detailed proof of the existence of equilibrium, and identify the "indifference threshold condition" as the key condition in the proof of existence.

Atkeson (2000) challenged the approach by Morris and Shin (2000), on the basis that it would not resist the introduction of prices. If rational individuals can learn about others' private information from asset prices, then the introduction of private noise may not solve the multiplicity problem. Angeletos and Werning (2006) formally address this issue, setting the conditions for uniqueness. Angeletos and Werning (2006) consider noncontinuous payoff

[1] Actions are strategic complements when the marginal payoff of a specific player increases with the actions of other players.

[2] We consider normally distributed signals, and this is why we do not present the work by Morris and Shin (1998). Yet, we do examine the main ideas in their contribution.

functions, which are useful to model events like speculative attacks and default. As a result of this assumption, one needs to consider the "state threshold condition" in the proof of existence of equilibrium.

Morris and Shin (2002) introduce two innovations. First, they use payoff functions which penalize extreme actions by individuals, thereby obtaining uniqueness in the perfect information case. Second, they assess the normative implications of the theory, and conclude that more public information may reduce social welfare. Unlike Morris and Shin (2002), Angeletos and Pavan (2004) consider payoff functions with complementarities at the social level, and for which better public information always increases welfare.

Our presentation is self-contained, and it includes the details of the mathematical derivations - which are often obscure in the original papers. These features make our paper a useful and unique guide for initial researchers in the field.

This paper is organized as follows. In the next section, we analyze the role of private and public information in the determination of equilibrium. In Section 3, we introduce market prices in the analysis. In Section 4, we assess the impact of information on welfare, when complementarity is only present at the private level. In Section 5, we perform the same exercise when there are complementarities at the aggregate level. Section 6 concludes our analysis.[3]

2 Rethinking Multiple Equilibria

Morris and Shin (2000), in their paper *Rethinking Multiple Equilibria in Macroeconomic Modelling*, examine the theoretical basis underlying the multiple equilibria literature in macroeconomics.

The existence of multiple equilibria derives from two common modeling assumptions. First, with strategic complementarity, agents wish to coordinate their actions. Second, economic fundamentals are common knowledge. Since Nash equilibria and rational expectation models presume that individuals correctly anticipate each other's actions, it follows that individual actions and beliefs can be perfectly coordinated in a way that creates multiple equilibria.

Multiplicity is a result of the indeterminacy in beliefs, since any of the equilibrium outcomes can be supported by a specific set of beliefs. In this context, equilibrium may be

[3] An appendix with proofs of some results is available at http://www.fep.up.pt/docentes/jjorge/research/appendix2.pdf.

determined by "sunspots" which define the set of self-fulfilling beliefs.

One would expect that strong fundamentals generate optimism which, in turn, causes good outcomes. Yet, models with "sunspots" do not describe how beliefs are formed, and how they shift.[4] Hence, models with multiple equilibria offer no rationale as to why good outcomes should be correlated with strong fundamentals, as they fail to link economic sentiment with the strength of the fundamentals.

Morris and Shin (2000) encourage a re-examination of the hypothesis of common knowledge about the economic fundamentals. They remove common knowledge by introducing noisy private signals which create uncertainty about others' actions. Since the actions of other players are not completely known, there cannot be perfect coordination. Their paper shows that there is a unique equilibrium when private information is sufficiently precise.

Because of uniqueness, it is possible to perform comparative statics analysis and policy recommendations, which is hard to do in models with multiple equilibria.

Morris and Shin (2000) also address one of the key questions in the literature on financial crises. Are crises triggered by fundamentals, or come as a result of panic? They suggest that weak economic fundamentals are more likely to trigger financial crises, highlighting the role of beliefs - economic agents tend to be more pessimistic about others actions when economic fundamentals are weak.

The paper presents an application to a bank run model, in line with Diamond and Dybvig (1983)'s model.

2.1 The Model

There are three dates $t \in \{0, 1, 2\}$ and a continuum of consumers, each endowed with 1 unit of consumption good. Consumption happens at either date 1 or 2, and there is a measure 1 of patient consumers who may consume on both dates and for whom goods are perfect substitutes, and a measure Λ of impatient consumers, who must consume at date 1. Ex ante (that is, at date 0), the probability of being impatient is given by $\frac{\Lambda}{1+\Lambda}$, and consumers learn their type at date 1.

The utility function of impatient consumers is given by $u_{imp}(c_1) = \ln c_1$, while the utility of patient consumer is $u_{pat}(c_1 + c_2) = \ln(c_1 + c_2)$. The values c_1 and c_2 refer to consumption at $t = 1$ and $t = 2$.

[4] These models often assume that strong fundamentals generate optimism but, theoretically, optimism is also conceivable with weak fundamentals.

At date 0, consumers can either store their endowment of the consumption good, or deposit it in the bank. Those consumers who have deposited their money in the bank at date 0, have to make a decision when they learn their type: they can leave their money in the bank, or withdraw.

The bank can invest the funds received at date 0 in cash, or in an illiquid technology that is only available to the bank. This technology is illiquid because it can be prematurely liquidated at a cost. The gross rate of return on this technology equals $R \cdot e^{-l}$ at date 2. The parameter $R > 1$ represents the maximum attainable return and e^{-l} represents the cost of premature liquidation. Variable l represents the proportion of the resources invested in the illiquid technology which are withdrawn at date 1. Hence $0 \leq l \leq 1$.[5] Let $\theta \equiv \ln R$, and the gross rate of return equals $e^{\theta - l}$. Assume $0 < \theta < 1$.[6]

Optimal Contract. The optimal contract defines the levels of consumption at both dates. The aim is to maximize the ex ante expected utility $\frac{\Lambda}{1+\Lambda} u_{imp}(c_1) + \frac{1}{1+\Lambda} u_{pat}(c_2)$, where we have assumed that patient consumers want to consume in the final period as it is optimal to benefit from the productive technology. Maximization is subject to the budget constraint $\Lambda c_1 + \frac{c_2}{R} \leq 1 + \Lambda$. The optimal contract defines $c_1 = 1$ and $c_2 = R$.

It can be easily verified that these quantities satisfy the incentive compatibility constraint $u_{pat}(c_1) \leq u_{pat}(c_2)$. Although patient consumers can withdraw 1 unit of good at date 1, they prefer to get R units of the consumption good at the final date.

Multiple equilibria. As shown by Diamond and Dybvig (1983), the optimal contract is vulnerable to bank runs and there are multiple equilibria. At date 1, impatient consumers have as a dominant strategy to withdraw and patient consumers play a coordination game with two equilibria.

For patient consumers, the utility of withdrawing at date 1 equals $u_{pat}(c_1) = \ln 1 = 0$. If a proportion l of patient consumers withdraw at date 1, the monetary payoff associated to those who leave their money in the bank will be $e^{\theta - l}$ and their utility will be $\theta - l$.

If a patient consumer expects that all other patient consumers do not withdraw, then it is optimal to maintain its wealth in the bank. In this case, $l = 0$ and its utility equals $\theta > 0$. If he expects that all other patient consumers withdraw, $l = 1$, then its utility is $\theta - 1 < 0$.

There is no Nash equilibrium with $0 < l < 1$, as some patient consumers would not be

[5] It is not very appealing that, if all resources invested in the illiquid technology are liquidated, the return in date 2 is positive ($R \cdot e^{-1} > 0$).

[6] Alternatively, one could think that the productive technology requires an investment at $t = 1$, and has a return equal to $e^{\theta - l}$ at the final date. In this case, $1 - l$ represents the amount of investment it the productive technology at $t = 1$. This technology can be seen as having increasing returns to scale.

playing their best response. Hence, there are two equilibria. In one equilibrium, all patient consumers maintain their money in the bank and, in the other, they all withdraw.

2.2 The Noisy Signal Eliminates Multiplicity

The next step is to introduce uncertainty regarding the fundamental θ. It should be noted that the optimal quantities $c_1 = 1$ and $c_2 = R$ derived in the previous section need not be optimal in the new setup. Nonetheless, Morris and Shin impose the benchmark contract in the new environment.

Suppose θ is a normal random variable with mean $\overline{\theta}$ and variance $1/\alpha$. This distribution is common knowledge, and this information is called public information. The precision of public information is given by parameter α. Variable θ can be represented as

$$\theta = \overline{\theta} + \frac{1}{\sqrt{\alpha}}\varepsilon \tag{1}$$

where ε is a standard normal random variable. Let $0 < \overline{\theta} < 1$, otherwise patient consumers have a strictly dominant strategy.

The introduction of uncertainty in public information does not change the results significantly, as we would still have multiple equilibria in this case. The real innovation of this contribution is the introduction of idiosyncratic uncertainty as follows.

Depositor i observes a noisy private signal $x_i = \theta + \frac{1}{\sqrt{\beta}}\varepsilon_i$, with ε_i following a standard normal distribution, independent across investors and independent of θ (and ε). This information is called private information and has precision β.

At date 1, the patient consumer i observes the private signal x_i (in the language of game theory, this is his type), updates his belief about θ, and decides if he withdraws or not.

A strategy is a rule of action which defines an action conditional on the observed signal. A profile of strategies is an equilibrium if, conditional on the available information to each depositor i, and given the strategies followed by other depositors, the action performed by each depositor i maximizes his conditional expected utility. In other words, we are looking for the Bayes Nash equilibrium of an imperfect information game.

For simplicity, assume that if withdrawing yields the same utility as leaving money in the bank, then the depositor chooses to leave the money in the bank. As the fundamental θ and the signal x_i are normally distributed, the depositor i's posterior belief about θ follows a normal distribution with mean

$$E\left[\theta|x_i\right]=\frac{\alpha\overline{\theta}+\beta x_i}{\alpha+\beta} \tag{2}$$

and variance

$$Var\left[\theta|x_i\right]=\frac{1}{\alpha+\beta}. \tag{3}$$

Let $\rho_i \equiv E\left[\theta|x_i\right]$. The conditional expectation ρ_i is a weighted average of $\overline{\theta}$ and x_i, in which the weights depend on the precision of public and private information. The conditional variance is constant and does not depend on the realizations of the fundamental and of the private signal. The above expressions derive from the following statistical result.

Lemma 1 *Let y and x be represented as a bivariate normal distribution*

$$\left[\begin{array}{c} y \\ x \end{array}\right] \sim N\left(\left[\begin{array}{c} E_y \\ E_x \end{array}\right],\left[\begin{array}{cc} V_y & V_{yx} \\ V_{xy} & V_x \end{array}\right]\right).$$

Then $E\left[y|x\right]=E_y+\frac{V_{yx}}{Vx}\left(x-E_x\right)$ *and* $Var\left[y|x\right]=V_y-\frac{V_{yx}^2}{Vx}$.

In our case, θ replaces y, and x_i replaces x. The unconditional mean of the signal x_i is $\overline{\theta}$, its unconditional variance is $\frac{1}{\alpha}+\frac{1}{\beta}$, and the covariance between x_i and θ equals $E\left[\left(x_i-\overline{\theta}\right)\left(\theta-\overline{\theta}\right)\right]=E\left[\left(\theta+\frac{1}{\sqrt{\beta}}\varepsilon_i-\overline{\theta}\right)\left(\theta-\overline{\theta}\right)\right]=\frac{1}{\alpha}$. Then $E\left[\theta|x_i\right]=\overline{\theta}+\frac{\frac{1}{\alpha}}{\frac{1}{\alpha}+\frac{1}{\beta}}\left(x_i-\overline{\theta}\right)=\frac{\alpha\overline{\theta}+\beta x_i}{\alpha+\beta}$ and $Var\left[\theta|x_i\right]=\frac{1}{\alpha}+\frac{\frac{1}{\alpha^2}}{\frac{1}{\alpha}+\frac{1}{\beta}}=\frac{1}{\alpha+\beta}$.

Let $\Phi\left(.\right)$ be the cumulative standard normal distribution function, and $\gamma=\frac{\alpha^2(\alpha+\beta)}{\beta(\alpha+2\beta)}$. There is a unique equilibrium when γ is small enough.

Proposition 1 *Provided that $\gamma \leq 2\pi$, there is a unique equilibrium. In this equilibrium, every patient consumer i withdraws if and only if $\rho_i<\rho^*$ where ρ^* is the unique solution to*

$$\rho^*=\Phi\left(\sqrt{\gamma}\left(\rho^*-\overline{\theta}\right)\right). \tag{4}$$

All patient consumers follow a switching strategy around the threshold ρ^*. Each patient consumer will withdraw his money whenever its expectation ρ_i falls below the threshold ρ^*. Models with common knowledge are characterized by "bang-bang" solutions, in which all patient consumers take the same decision. This is (generally) not true with imperfect information. With noisy private signals, some depositors maintain their money at the bank while others withdraw. There are always inefficiencies in equilibrium because some depositors make the wrong decisions about withdrawing or not their money.

The proof of Proposition 1 has two parts - proving uniqueness and showing existence. We start by showing that the above equilibrium indeed exists.

The indifference threshold condition. A patient consumer whose posterior belief about θ equals ρ^* ought to be indifferent between leaving his money in the bank and withdrawing it. The utility from withdrawing is zero, while the utility of leaving money deposited in the bank equals $\theta - l$. Hence, the threshold ρ^* satisfies

$$E\left[\theta - l|\rho^*\right] = 0. \tag{5}$$

Since $\rho_i = \frac{\alpha\overline{\theta}+\beta x_i}{\alpha+\beta}$, conditioning on ρ_i or conditioning on x_i is equivalent. Hence, $E\left[\theta - l|\rho^*\right] = E\left[\theta|\rho^*\right] - E\left[l|\rho^*\right] = \rho^* - E\left[l|\rho^*\right]$. It remains to check the value of $E\left[l|\rho^*\right]$. The noise ε_i is independent of θ and, as a result, the expected proportion of patient consumers who withdraw is equal to the probability that any particular patient consumer withdraws. Hence, $E\left[l|\rho^*\right] = prob\left[\rho_j < \rho^*|\rho^*\right]$, since every patient consumer follows a switching strategy around ρ^*.

As an intermediate step, we compute the probability that depositor i attaches to some other depositor j having a posterior belief ρ_j lower than himself, that is $prob\left[\rho_j < \rho_i|\rho_i\right]$. Since conditioning on ρ_i or conditioning on x_i is equivalent, then the posterior distribution of the fundamental θ is normal with mean and variance given by (2) and (3). Moreover, $\rho_j < \rho_i \Leftrightarrow \frac{\alpha\overline{\theta}+\beta x_j}{\alpha+\beta} < \rho_i \Leftrightarrow x_j < \rho_i + \frac{\alpha}{\beta}\left(\rho_i - \overline{\theta}\right)$. Hence, $prob\left[\rho_j < \rho_i|\rho_i\right] = prob\left[x_j < \rho_i + \frac{\alpha}{\beta}\left(\rho_i - \overline{\theta}\right)|\rho_i\right]$.

Since $x_j = \theta + \frac{1}{\sqrt{\beta}}\varepsilon_j$, the distribution of x_j conditional on ρ_i is normal with mean ρ_i and precision $\frac{\beta(\alpha+\beta)}{\alpha+2\beta}$. This is because, according to Lemma 1, $E\left[x_j|\rho_i\right] = E\left[x_j|x_i\right] = \overline{\theta} + \frac{Cov[x_i,x_j]}{Var[x_i]}\left(x_i - \overline{\theta}\right) = \overline{\theta} + \frac{\frac{1}{\alpha}}{\frac{1}{\alpha}+\frac{1}{\beta}}\left(x_i - \overline{\theta}\right) = \rho_i$ and $Var\left[x_j|\rho_i\right] = Var\left[x_j|x_i\right] = Var[x_j] - \frac{Cov^2[x_i,x_j]}{Var[x_i]} = \left(\frac{1}{\alpha} + \frac{1}{\beta}\right) - \frac{\left(\frac{1}{\alpha}\right)^2}{\frac{1}{\alpha}+\frac{1}{\beta}} = \frac{\alpha+2\beta}{\beta(\alpha+\beta)}$. Hence, we can compute $prob\left[\rho_j < \rho_i|\rho_i\right]$ as

$$prob\left[x_j < \rho_i + \tfrac{\alpha}{\beta}\left(\rho_i - \overline{\theta}\right)|\rho_i\right] =$$

$$prob\left[\frac{x_j - \rho_i}{\sqrt{\frac{\alpha+2\beta}{\beta(\alpha+\beta)}}} < \frac{\rho_i + \frac{\alpha}{\beta}\left(\rho_i - \overline{\theta}\right) - \rho_i}{\sqrt{\frac{\alpha+2\beta}{\beta(\alpha+\beta)}}}|\rho_i\right] =$$

$$= \Phi\left(\sqrt{\frac{\beta(\alpha+\beta)}{\alpha+2\beta}}\frac{\alpha}{\beta}\left(\rho_i - \overline{\theta}\right)\right) = \Phi\left(\sqrt{\gamma}\left(\rho_i - \overline{\theta}\right)\right).$$

We are now ready to compute $E\left[l|\rho^*\right]$. For an individual with expectation ρ^*, we have $E\left[l|\rho^*\right] = \Phi\left(\sqrt{\gamma}\left(\rho^* - \overline{\theta}\right)\right)$. We can go back to equation (5) and substitute the results obtained so far , and get $\rho^* - \Phi\left(\sqrt{\gamma}\left(\rho^* - \overline{\theta}\right)\right) = 0$. The value ρ^* is the point at which the line $\Phi\left(\sqrt{\gamma}\left(\rho - \overline{\theta}\right)\right)$ crosses the 45^o degrees line, and this point always exists. Hence, equilibrium always exists.

Uniqueness. The slope of $\Phi\left(\sqrt{\gamma}\left(\rho-\overline{\theta}\right)\right)$ equals $\sqrt{\gamma}\phi\left(\sqrt{\gamma}\left(\rho-\overline{\theta}\right)\right)$, where $\phi\left(.\right)$ is the density of the standard normal distribution function. Since the maximum value of the density function is $\frac{1}{\sqrt{2\pi}}$, the slope of $\Phi\left(\sqrt{\gamma}\left(\rho-\overline{\theta}\right)\right)$ is inferior or equal to $\sqrt{\frac{\gamma}{2\pi}}$. Since $\gamma\leq 2\pi$, there can be at most one point of intersection.

Hence, there is one single equilibrium in the class of switching strategies. Yet, there may exist other equilibria in other classes of strategies. Following Carlsson and van Damme (1993), Morris and Shin (2000) show that the proposed equilibrium is the only one to survive the iterated deletion of strictly dominated strategies. This makes their argument quite strong, as this equilibrium concept is stronger than the definition of Nash Equilibrium. We do not show their argument here, as the proof of uniqueness is fairly clear in the original paper.

The precision of private information. To obtain a unique equilibrium, private information must be sufficiently precise, so that parameter γ is low enough. When private information is sufficiently precise, there is enough diversity in beliefs such that individuals decide based on their own private signals, and are unable to coordinate their actions.

When public information is too precise (α is high) we are in a situation akin to the traditional case without private information. The traditional case can be seen as the extreme case in which the *relative* precision of public information is infinitely high (even though the *absolute* precision of public information might be low). In the traditional case, all information is common knowledge and public information allows agents to coordinate their actions and beliefs in a way which invites multiplicity of equilibria. The same happens when private information is imprecise.

Morris and Shin did not provide an interpretation for private information. Later, Angeletos and Pavan (2004) remarked that the private signal x_i introduces idiosyncratic variation in market expectations about the fundamentals, and thus can be interpreted as heterogeneity in the filtering and interpretation of publicly available information.

2.3 Comparative statics and policy analysis

Unlike models with multiple equilibria, it is possible to perform secure comparative statics analysis in models with a unique equilibrium. As a result, in these models it is possible to make sound policy recommendations. More specifically, Morris and Shin (2000) study the implementation of an early-withdrawal penalty t imposed on consumers who withdraw at date 1.

Morris and Shin (2000) apply a technique that has become popular in the literature, and which serves two purposes. First, it simplifies the mathematical derivations and allows one to measure results easily. Second, it allows one to compare directly the certainty case with the uncertainty case. They focus on what happens in the limit when the private signals of depositors become very precise, and noise becomes negligible. This corresponds to the case where $\beta \to \infty$, in which case depositors know the realization of the fundamental θ because $x_i \to \theta$ and $\rho_i \to \theta$. In this case $\gamma \to 0$, and from expression (4) we obtain $\rho^* = \frac{1}{2}$.

This result may seem surprising. Although there is (almost) certainty about the fundamental θ, all patient consumers withdraw their money from the bank when $0 < \theta < \frac{1}{2}$, even if they would be better by not doing so. Despite (almost) perfect information, they are unable to coordinate their decisions. The key to understanding this result is to note that *strategic uncertainty* (that is, uncertainty about others' actions) is not resolved even when the private signal becomes very precise. Private noise destroys common knowledge, thereby preventing coordination.

The case in which noise becomes negligible can be seen as a small perturbation of the certainty case. The lesson is clear. Multiplicity of equilibria in the original bank run game does not resist the introduction of an arbitrarily small amount of noise.

Going back to the analysis of the implementation of an early-withdrawal penalty, it implies that withdrawal at date 1 yields utility $\ln(1-t)$ and expression (5) would become $E[\theta - l|\rho^*] = \ln(1-t)$. This implies that the new critical threshold would be $\rho^* = \frac{1}{2} + \ln(1-t)$. As the new threshold is lower than $\frac{1}{2}$, the penalty increases efficiency for those states in which $\theta \in \left[\frac{1}{2} + \ln(1-t), \frac{1}{2}\right)$.

2.4 Observable implications

The model also has testable implications. First, each patient consumer will withdraw his money whenever its expectation ρ_i falls below the threshold ρ^*. Since private signals are positively correlated with the realization of the fundamental θ, the size of the bank run will be negatively correlated with the value of the fundamental. The incidence of early-withdrawals among patient consumers is given by l, and depends on the realization of the fundamental. A patient consumer i withdraws whenever $\frac{\alpha\overline{\theta}+\beta x_i}{\alpha+\beta} < \rho^*$. Rearranging terms, he will withdraw whenever its private signal falls below a threshold x^*, that is whenever $x_i < x^* \equiv \frac{\alpha+\beta}{\beta}\rho^* - \frac{\alpha}{\beta}\overline{\theta}$. Since $x_i = \theta + \frac{1}{\sqrt{\beta}}\varepsilon_i$, the probability that patient consumer i withdraws conditional on the realization of the fundamental θ is equal to $prob\,[x_i < x^*|\theta] = prob\left[\frac{x_i-\theta}{\frac{1}{\sqrt{\beta}}} < \frac{x^*-\theta}{\frac{1}{\sqrt{\beta}}}|\theta\right] = \Phi\left(\sqrt{\beta}\,(x^*-\theta)\right)$. With a continuum of patient consumers, the law of large numbers allows us to treat aggregate variables as deterministic. Hence, the incidence of withdrawals by patient consumers conditional on the realization of the fundamental θ is equal to $l(\theta) = prob[x_i < x^*|\theta] = \Phi\left(\sqrt{\beta}(x^* - \theta)\right)$. Clearly, the incidence of early-withdrawals is high when the realization of the fundamental θ is low.

Second, a shift in the expected realization $\overline{\theta}$, implies a shift in the threshold ρ^*. To see this, apply the implicit function theorem to expression (5) to get $\frac{\partial \rho^*}{\partial \overline{\theta}} < 0$.

When public information states that economic fundamentals are weak (as measured by $\overline{\theta}$), the threshold ρ^* will be high and the incidence of early-withdrawals $l(\theta)$ increases for all realizations of θ.

3 Crises and Prices

In the previous analysis, individuals have no information other than the prior distribution of θ and their own private signal to consider when they decide whether to withdraw or not.[7] Atkeson (2000) points out that this would change if one introduced markets and prices into the model.

Rational individuals can learn about others' private information from asset prices. Consider the existence of bank equity being traded before depositors take their decisions. This asset should be priced in equilibrium in a way that accurately reflects its subsequent return. The price of bank equity should reveal the incidence of early-withdrawls, since the value of l is pinned down in equilibrium. Bank equity will have a low price when the fundamental θ is low (and most depositors withdraw their money from the bank), and a high price when θ is high. Hence, the price reveals the state of the world θ and, as a result, we obtain multiple equilibria.

Angeletos and Werning (2006), in their paper *Crises and Prices: Information Aggregation, Multiplicity and Volatility*, address the informational role of prices in the context of global games. They introduce a financial market in a coordination game. Prices aggregate dispersed private information, and act as an endogenous public signal about the economic fundamental.

The precision of endogenous public information increases with the precision of exogenous private information and, as result, lower levels of private noise do not necessarily contribute towards uniqueness.

In their paper, Angeletos and Werning (2006) revisit the model of currency crises by Morris

[7] Hence, individuals cannot observe the queues forming in front of a bank, as this would give them an idea about the value of l.

and Shin (1998). Yet, it is also possible to reinterpret their model as a model of bank runs.

3.1 The Model

There is a continuum of agents $i \in [0,1]$, and each agent can choose between two actions, either attack $a_i = 1$, or not attack $a_i = 0$. The attack action replaces the withdraw action in Morris and Shin (2000).

The payoff from not attacking is normalized to zero. The payoff from attacking is $1-c$ if the the status quo is abandoned (that is, the attack succeeds), and $-c$ otherwise. The value of $c \in (0,1)$ parametrizes the cost of attacking.

The attack succeeds if and only if $A > \theta$, where A represents the mass of agents attacking and θ denotes the strength of the status quo. Hence, the payoff function can be represented as:

$$u\left(a_i, A, \theta\right) = \begin{cases} 1-c & if \quad a_i = 1 \text{ and } A > \theta \\ -c & if \quad a_i = 1 \text{ and } A \leq \theta \\ 0 & if \quad a_i = 0. \end{cases}$$

This is a coordination game because $u\left(1, A, \theta\right) - u\left(0, A, \theta\right)$ increases with A, so that the incentives to attack increase with the mass of agents attacking.

Exogenous public information Angeletos and Werning (2006) start by assuming that public information about the economic fundamental θ is exogenous. Individuals have a common prior about θ which follows a uniform distribution over the entire real line. Agents receive an exogenous public signal $z = \theta + \frac{1}{\sqrt{\alpha}}\varepsilon$ where ε is a standard normal random variable. Put differently, we can rewrite this expression as $\theta = z - \frac{1}{\sqrt{\alpha}}\varepsilon$, which is similar to expression (1) with z replacing the mean $\overline{\theta}$. It follows that the definition of exogenous public information in Angeletos and Werning (2006) is equivalent to the definition of public information in Morris and Shin (2000).

Like in Morris and Shin (2000), each individual i observes a private signal $x_i = \theta + \frac{1}{\sqrt{\beta}}\varepsilon_i$, where ε_i is standard normal, independent and identically distributed across individuals, and independent of ε. It is easy to show that $E\left[\theta | x_i\right] = \frac{\alpha z + \beta x_i}{\alpha + \beta}$ and $Var\left[\theta | x_i\right] = \frac{1}{\alpha+\beta}$.

The definition of equilibrium is very similar to Morris and Shin (2000). Yet, the results differ in dimensions that are worth mentioning.

Proposition 2 *In the game with exogenous information, the equilibrium is unique if and only* *if private noise is small relative to public noise, so that* $\frac{\alpha}{\sqrt{\beta}} \leq \sqrt{2\pi}$.

For sufficiently precise private information (β large enough) we obtain uniqueness. Angeletos and Werning (2006) focus on switching strategies in which the individual attacks if and only if his private signal x_i is lower than a threshold x^*. It is straightforward to derive the equilibrium in terms of switching strategies around a threshold ρ^* (as Morris and Shin, 2000 did), given the equivalence between the signal x_i and the posterior ρ_i.

To prove the existence of equilibrium, two conditions are needed: the indifference threshold condition and the state threshold condition. We start with the latter.

The state threshold condition. The status quo is about to change when $A = \theta$, where A is the mass of attackers who have received a signal below the marginal signal x^*. Given that there is a continuum of individuals, A is equal to the probability that an individual receives a signal lower than x^*. Denote by θ^* the critical state θ which separates those states in which the status quo is abandoned from those states in which it is not, that is $A = \theta^*$. When $\theta = \theta^*$, the distribution of x_i is normal with mean θ^* and precision β. Hence, $A = \Phi\left(\sqrt{\beta}\,(x^* - \theta^*)\right)$. Since $A = \theta^*$, we obtain

$$\Phi\left(\sqrt{\beta}\,(x^* - \theta^*)\right) = \theta^* \Leftrightarrow x^* = \theta^* + \frac{\Phi^{-1}(\theta^*)}{\sqrt{\beta}}. \tag{6}$$

The indifference threshold condition. Angeletos and Werning (2006) derive the indifference threshold condition in terms of the private signal x_i (and not in terms of the posterior ρ_i, as Morris and Shin, 2000 did). At the critical threshold x^*, the individual is indifferent between attacking and not attacking. Not attacking yields zero, while the individual's expected payoff from attacking depends on the subjective probability that the status quo is abandoned, as well as on the returns when the attack succeeds and does not succeed. Since the attack succeeds whenever $\theta < \theta^*$, individual i attacks whenever $\mathit{prob}\,[\theta < \theta^*|x_i]\,(1-c) + \mathit{prob}\,[\theta \geq \theta^*|x_i]\,(-c) > 0$. Individual i believes that θ has a normal distribution with mean $\frac{\alpha z + \beta x_i}{\alpha+\beta}$ and precision $\alpha+\beta$. Hence, the probability that individual i attributes to a change in the status quo is equal to $\mathit{prob}\,[\theta < \theta^*|x_i] = \Phi\left(\sqrt{\alpha+\beta}\left(\theta^* - \frac{\alpha z + \beta x_i}{\alpha+\beta}\right)\right)$. For the marginal individual who is indifferent between attacking or not,

$$\begin{aligned} &\Phi\left(\sqrt{\alpha+\beta}\left(\theta^* - \frac{\alpha z + \beta x_i}{\alpha+\beta}\right)\right)(1-c) \\ &-\left[1 - \Phi\left(\sqrt{\alpha+\beta}\left(\theta^* - \frac{\alpha z + \beta x_i}{\alpha+\beta}\right)\right)\right] c \quad = \quad 0 \end{aligned}$$

This indifference condition applies for an individual with a signal equal to x^*, and can be rewritten as

$$\theta^* - \frac{\alpha z + \beta x^*}{\alpha + \beta} = \frac{\Phi^{-1}(c)}{\sqrt{\alpha + \beta}} \tag{7}$$

Critical value of θ. Equilibrium is identified with the joint solution to the threshold conditions (6) and (7). Substituting (6) into (7) yields $\theta^* = \Phi\left(\frac{\alpha}{\sqrt{\beta}}\left(-\frac{\sqrt{\alpha+\beta}}{\alpha}\Phi^{-1}(c) + (\theta^* - z)\right)\right)$ or, equivalently,

$$\theta^* = \Phi\left(\frac{\alpha}{\sqrt{\beta}}\left(\frac{\sqrt{\alpha+\beta}}{\alpha}\Phi^{-1}(1-c) + (\theta^* - z)\right)\right) \tag{8}$$

because $\Phi(1-c) = -\Phi(c)$. Equation (8) has a unique solution, if the expression in the right hand side has slope less than one everywhere. Since the slope of the cumulative standard normal distribution is given by the corresponding density function (which is always inferior to $\frac{1}{\sqrt{2\pi}}$), a sufficient condition for a unique solution for θ^* is given by $\frac{\alpha}{\sqrt{\beta}} \leq \sqrt{2\pi}$.

Differences between the Morris and Shin (2000) and Angeletos and Werning (2006) models. The main difference is the definition of the payoff functions. In Morris and Shin (2000) the payoff on the bank deposit depends on the incidence of early withdrawals (and is, thus, a continuous variable), whereas in Angeletos and Werning (2006) the payoff depends on whether the status quo is abandoned or not (and is, thus, a discrete variable).

This difference has two important consequences for the proof of existence of equilibrium. On the one hand, obtaining the indifference threshold condition is considerably more challenging in the Morris and Shin (2000) setup. On the other hand, one must estimate the state threshold condition in the Angeletos and Werning (2006) framework.

3.2 Prices as endogenous sources of public information

Assume there is a financial market in which agents trade an asset prior to playing the coordination game. The return on this asset depends on the fundamental θ, and individuals trade based on their private signals. As a result, the equilibrium price will convey information that is useful in the coordination game.

The fundamental θ is drawn from a uniform distribution over the real line, and each individual i receives the exogenous private signal x_i. For tractability reasons we separate the coordination game from the trading in the financial market. Despite having two stages, the game is not dynamic from the strategic point of view, and agents can be seen as interacting in two separate stages.

In the first stage, individuals trade the risky asset at price p. For simplicity assume that the asset's return equals θ. We adopt the CARA-normal framework introduced by

Grossman and Stiglitz (1976). The utility of individual i is $V(w_i) = -e^{-\gamma w_i}$ for $\gamma > 0$, where $w_i = w_0 + (\theta - p) k_i$ is the final wealth, w_0 is the initial endowment, and $k_i \in \mathbb{R}$ is investment in the risky asset. The supply of the asset is uncertain and not observed, given by $K^S(\varepsilon_s) = \sigma_s \varepsilon_s$, with $\sigma_s > 0$, and ε_s is a standard normal random variable, independent of θ, ε, and the private noise ε_i. This formulation means that the risky asset exists in zero net supply plus some noise - parametrized by σ_s - which prevents a fully revealing equilibrium.

The second stage is essentially the same as the model in Section 3.1. Individuals decide whether to attack or not, the status quo changes if $A > \theta$, and the payoffs equal $u(a_i, A, \theta)$. There is an important difference in the second stage with respect to the model in Section 3.1. Individuals observe the price that cleared the financial market, and the endogenous price p replaces the exogenous public signal z. The eventual change in the status quo, the asset's return and the payoffs from both stages are realized at the end of the second stage.

Individual demand of the risky asset $k(x,p)$ is a function of x and p, the realizations of the private and public signals; aggregate demand $K(\theta, p)$ is a function of θ and p. The individual decision to attack or not $a(x,p)$ is a function of x and p; the mass of agents attacking $A(\theta, \rho, p)$ is a function of θ and p. The equilibrium price depends on the realizations of the fundamental θ and ε_s; define the equilibrium price function as $P(\theta, \varepsilon_s)$.

Definition 1 *An equilibrium is a price function $P(\theta, \varepsilon_s)$, individual strategies for investment in the risky asset $k(x,p)$, and individual strategies about whether to attack or not $a(x,p)$, such that:*

$k(x,p) \in \arg\max_{k \in \mathbb{R}} E[V(w_0 + (\theta - p) k) | x, p]$;

$K(\theta, p) = E[k(x,p) | \theta, p]$;

$P = P(\theta, \varepsilon_s)$ *is the solution to* $K(\theta, P) = K^S(\varepsilon_s)$;

$a(x,p) \in \arg\max_{a \in \{0,1\}} E[u(a, A(\theta, p), \theta) | x, p]$;

$A(\theta, p) = E[a(x,p) | \theta, p]$;

and the status quo is abandoned if and only if $A(\theta, P) > \theta$.

The above conditions define, respectively, the individual demand, the aggregate demand, and the market clearing for the risky asset, the individual attack strategy, and the mass of agents attacking. These conditions define a rational expectations competitive equilibrium for the first stage, and a Bayesian equilibrium for the second stage.

Equilibrium. In the first stage, we guess a linear price function. Observing the price realization then is equivalent to observing a normally distributed signal with some precision $\alpha_p > 0$. The posterior of θ conditional on x and p is normally distributed with mean $\frac{\alpha_p p+\beta x_i}{\alpha_p+\beta}$ and precision $\alpha_p + \beta$. Individuals choose the amount of the risky asset which maximizes their expected utility $E\left[V\left(w_i\right)|x_i, p\right] = E\left[-e^{-\gamma w_i}|x_i, p\right] = E\left[-e^{-\gamma w_0 - \gamma(\theta-p)k_i}|x_i, p\right]$. In order to proceed, the following statistical result is needed.

Lemma 2 ***(Moment-generating function of a normal random variable).*** *When t is a constant and y has a normal distribution with mean μ and variance ν, then $E\left[e^{ty}\right] = e^{t\mu+\frac{t^2}{2}\nu}$.*

Applying the above result, we obtain $E\left[V\left(w_i\right)|x_i, p\right] = E\left[-e^{-\gamma w_0 - \gamma(\theta-p)k_i}|x_i, p\right] =$ $E\left[-e^{-\gamma w_0-\gamma\left(\frac{\alpha_p p+\beta x_i}{\alpha_p+\beta}-p\right)k_i+\frac{\gamma^2}{2}k_i^2\frac{1}{\alpha_p+\beta}}|x_i, p\right]$. Hence

$$\max_{\{k_i\}} E\left[V\left(w_i\right)|x_i, p\right] = \max_{\{k_i\}} \left(\frac{\alpha_p p + \beta x_i}{\alpha_p + \beta} - p\right) k_i - \frac{\gamma}{2} k_i^2 \frac{1}{\alpha_p + \beta}.$$

The first order condition yields an individual asset demand equal to $k\left(x, p\right) = \frac{\beta}{\gamma}\left(x - p\right)$. The aggregate demand of the risky asset equals $K\left(\theta, p\right) = E\left[k\left(x, p\right)|\theta, p\right] = \frac{\beta}{\gamma}\left(\theta - p\right)$. The market clearing condition implies $P\left(\theta, \varepsilon_s\right) = \theta - \frac{\gamma\sigma_s}{\beta}\varepsilon_s$, which verifies the initial guess with $\alpha_p = \left(\frac{\beta}{\gamma\sigma_s}\right)^2$. This result highlights the informative role of prices because the precision of public information improves with private information.

The second stage is equivalent to the benchmark model in the previous section, with the price p playing the role of the public signal z. Replace α with α_p, and the uniqueness condition becomes $\frac{\alpha_p}{\sqrt{\beta}} \leq \sqrt{2\pi} \Leftrightarrow \frac{\sqrt{\beta^3}}{\gamma^2\sigma_s^2} \leq \sqrt{2\pi}$. Hence we obtain the following proposition.

Proposition 3 *In the game with endogenous information, the equilibrium is unique if and only if $\frac{\sqrt{\beta^3}}{\gamma^2\sigma_s^2} \leq \sqrt{2\pi}$.*

In contrast with the exogenous information case expressed in Proposition 2, too precise private information implies the existence of multiple equilibria. The introduction of a financial market implies that better private information (i.e. large β) also improves the precision of public information at a faster rate, sufficient to ensure multiple equilibria. Yet, if the uncertainty σ_s associated with the supply of the risky asset is large enough, then the price is not sufficiently informative. In this case, individuals do not use the public signal coordinate their actions and we obtain a unique equilibrium.

3.2.1 Endogenous returns and price multiplicity

Motivated by the fact that crises affect asset market returns, Angeletos and Werning (2006) consider the case in which the risky asset's return is endogenously determined by the coordination game. The novel result is that multiplicity can also emerge in the financial price.

If the return in the first stage is endogenously determined by the coordination game, the overall effect of prices becomes more complex. A higher price realization in the first stage indicates a high realization of the fundamental θ, thereby making individuals less willing to attack in the coordination game. If a smaller attack raises the return on the asset traded in the first stage, then the demand for the asset can increase with its price and there is a backward-bending demand curve. With a backward-bending demand curve, it is possible to obtain multiple market clearing prices.

3.3 Applications

Morris and Shin (2004), in their paper *Coordination Risk and the Price of Debt*, apply the results obtained so far to study the coordination problem of creditors facing a borrower in distress. Creditors of a distressed firm face the same problem as depositors facing a bank run, and Morris and Shin (2004) explore the idea of solvent but illiquid borrowers as they had explored the idea of solvent but illiquid banks. The hypothesis is that the coordination problem impacts on the price of defaultable debt.

Morris and Shin (2006), in their paper *Catalytic Finance: When Does it Work?*, study the effects of lending by the International Monetary Fund (IMF) in the context of sovereign debt crises. They argue that the provision of official assistance to a sovereign with limited access to financial markets stimulates the private sector creditors into rolling over short term loans, thereby alleviating the funding crisis faced by the debtor country. Liquidity provision by an official institution like the IMF can work to prevent a destructive run by moving the critical threshold θ^*.

There are lessons to take from their methodological approach, as they use a dynamic model in which the sovereign, the IMF, and the individual short term investors move sequentially. Morris and Shin (2006) can easily solve this dynamic problem because short term investors move in the last stage. By applying backward induction, they can solve the global game component by taking the actions taken by the IMF and the sovereign government as exogenous.

Corsetti, Guimarães and Roubini (2006), in their paper *International Lending of Last*

Resort and Moral Hazard: A Model of IMF's Catalytic Finance, study a problem similar to Morris and Shin (2006) but adopt a different analytical framework. They consider a global game in which all players move simultaneously, and the IMF is a large player relative to the other creditors. The IMF then conditions the nature of the strategic interactions of the creditors, thereby affecting critical threshold θ^*.

4 Social Value of Public Information

In their paper *Social Value of Public Information*, Morris and Shin (2002) address two main issues: one methodological and one substantive. From a substantive viewpoint, they move beyond the study of the positive (or descriptive) implications of the theory, and consider the normative (or prescriptive) implications. Specifically, they assess the welfare implications of more precise information and find the best quantity of information.

Methodologically, Morris and Shin (2002) consider a combination of continuous strategy spaces and payoff functions for which there is a large cost associated with undertaking extreme actions. As a result, there is a unique equilibrium, even in the perfect information case.[8] This framework is well-suited to analyze problems for which multiplicity of equilibria is not an issue.

The main result of Morris and Shin (2002) is that, under some conditions, more public information is bad for social welfare. Yet, Svensson (2006) argues that this is unlikely to be the case for reasonable parameters.

4.1 The Model

There is a continuum of individuals, indexed by i, and uniformly distributed over the interval $[0,1]$. Each individual i chooses an action $a_i \in \mathbb{R}$, with $\mathbf{a}$ representing the action profile of all individuals. Individual i's payoff function is

$$u_i(\mathbf{a}, \theta) = -\left[(1-\lambda)(a_i - \theta)^2 + \lambda\left(L_i - \overline{L}\right)\right]$$

where θ is the fundamental (the underlying state), λ is a constant with $0 < \lambda < 1$, and $L_i = \int_0^1 (a_j - a_i)^2 \, dj$ and $\overline{L} = \int_0^1 L_j dj$. Function L_j measures the average distance between i's action and the action profile of the whole population, while $\overline{L}$ is an average of the average

[8] Formally, Morris and Shin (2002) consider a utility function which generates a linear first order condition in the individual maximization problem. There is a unique optimal decision for each individual, which depends on his individual information.

individual distances.

According to the above payoff function, individual i has two objectives. First, he wishes to take an action which is as close as possible to the underlying state θ; the first component of the payoff function is a quadratic loss in the distance between the fundamental θ and his action a_i.

Second, he wishes to take an action which is as close as possible to the actions of the other individuals (that is, he wants to set a low value for L_i). The term $\overline{L}$ standardizes the value of L_i; setting a_i very different from the actions of the whole population is not so detrimental for his utility, if everybody else makes the same mistake (that is, $\overline{L}$ is high). The second term in the payoff function is the Keynesian "beauty contest" component, which creates incentives for individuals to second-guess the actions of others. The parameter λ gives the weight on this second-guessing motive.

The social welfare is defined as the (normalized) average of individual utilities

$$\begin{aligned} W(\mathbf{a},\theta) &= \frac{1}{1-\lambda}\int_0^1 u_i(\mathbf{a},\theta)\,di = -\frac{1}{1-\lambda}\int_0^1 \left[(1-\lambda)(a_i-\theta)^2+\lambda\left(L_i-\overline{L}\right)\right]di \\ &= -\int_0^1 (a_i-\theta)^2\,di. \end{aligned} \tag{9}$$

Thus, the social planner wants to minimize the distance between agents actions and the economic fundamental θ.

The second-guess motive does not influence social welfare, since $\int_0^1\left(L_i-\overline{L}\right)di = 0$. Second-guessing has a zero-sum nature, as winners gain at the expense of the losers. But, since it influences individual decisions, the second-guessing motive creates a negative externality.

To see this, consider the first order condition of individual i, $\frac{\partial E[u_i(\mathbf{a},\theta)]}{\partial a_i}=0 \Leftrightarrow$

$E_i\left[-2(1-\lambda)(a_i-\theta)-\lambda\left(\frac{\partial L_i}{\partial a_i}-\frac{\partial \overline{L}}{\partial a_i}\right)\right]=0$, where $E_i[.]$ is the expectation conditional on the information of individual i.

Individual i is infinitesimal and takes $\overline{L}$ as a constant, so that $\frac{\partial \overline{L}}{\partial a_i}=0$.

Hence, $E_i\left[-(1-\lambda)(a_i-\theta)+\lambda\int_0^1(a_j-a_i)\,dj\right]=0$.

Define $\overline{a}=\int_0^1 a_j dj$ as the average action in the population, and we obtain

$$a_i=(1-\lambda)E_i[\theta]+\lambda E_i[\overline{a}]\,. \tag{10}$$

Hence, individual actions depend on the expected fundamental and on the expected actions of others.

The certainty case. When individuals know the value of the fundamental θ, the equilibrium is unique and entails $a_i = \theta$ for all i.[9] Social welfare is maximized at equilibrium.

Unlike in previous models, there is a unique (symmetric) equilibrium in the perfect information case. This is because of the way in which the authors design the payoff function $u_i(\mathbf{a}, \theta)$. There is a large cost associated with undertaking extreme actions, like $a_i \to -\infty$ or $a_i \to +\infty$, and all agents prefer to take "intermediate" actions. This feature prevents "bang-bang" solutions.

Equilibrium with public signals and perfect information. Morris and Shin (2002) consider the case where individuals face uncertainty and have access to exogenous public information which is common knowledge. Individuals have a common prior about θ which follows a uniform distribution over the entire real line. Individuals receive an exogenous public signal $z = \theta + \frac{1}{\sqrt{\alpha}}\varepsilon$ where ε is a standard normal random variable. Conditional on the public signal z, the fundamental θ has a normal distribution with mean z and precision α, that is, $\theta = z - \frac{1}{\sqrt{\alpha}}\varepsilon$. Using equation (10), we obtain a unique equilibrium given by $a_i(z) = z$, since $E_i[\theta] = E[\theta|z] = z$ and $E_i[\overline{a}] = E[\overline{a}|z] = E\left[\int_0^1 a_j(z)\, dj|z\right] = E\left[\int_0^1 z\; dj|z\right] = z$. Conditional on the fundamental θ, the expected social welfare is equal to

$$E[W(\mathbf{a}, \theta)|\theta] = E\left[-\int_0^1 (z-\theta)^2\, di|\theta\right] = -\int_0^1 E\left[(z-\theta)^2|\theta\right] di = -\frac{1}{\alpha}.$$

Hence, more precise information raises social welfare.

4.2 Equilibrium with private and public signals

Morris and Shin (2002) consider the information structure with exogenous public information and private signals that we have studied in Section 3.1. More specifically, each individual also receives a private signal $x_i = \theta + \frac{1}{\sqrt{\beta}}\varepsilon_i$, where ε_i is a standard normal random variable, independent and identically distributed across individuals, and independent of ε. Under this setup we obtain the following result.

Proposition 4 *There is a unique equilibrium, given by* $a_i = \frac{\alpha z + \beta(1-\lambda)x_i}{\alpha + \beta(1-\lambda)}$.

Individual actions depend on the available information. In the certainty case, individual actions equal the known fundamental. In the perfect information case, actions equal the

[9] It is easy to prove this statement by contradiction. If there was an equilibrium with $a_i \neq \theta$, individual i would want to set his action a little bit closer to θ.

public signal, which is the best available information about the fundamental. In the imperfect information case, individual actions are a weighted average of the two pieces of information received. But, in this case, individual actions are different from the best estimate of the fundamental (this would be $a_i = \frac{\alpha z + \beta x_i}{\alpha + \beta}$). This happens because of the second-guessing motive.

In the certainty and in the perfect information cases, individuals second-guess others' decisions correctly, so that the "beauty contest" term does not interfere with the individual optimal decision. All individuals end up choosing the same action, and $L_i - \overline{L} = 0$. This is no longer true in the imperfect information case. With private signals, individual actions are different across agents (so that $L_i - \overline{L} \neq 0$) and the second-guessing motive influences individual decisions. The individual decision becomes $a_i = \frac{\alpha z + \beta(1-\lambda) x_i}{\alpha + \beta(1-\lambda)}$ and, since the weight of the second-guessing motive is given by λ, it is easy to see that the weight of the second-guessing motive reduces the importance of private information x_i on individual decisions.

Second-guessing implies that individuals give more weight to public information than to private information. As a result, there is a disproportionate influence of the public signal z in influencing the individuals' actions. The magnitude of this effect is greater when λ is large.

In order to prove the existence of equilibrium, we guess that all individuals follow a linear strategy

$$a_i(z, x_i) = \kappa x_i + (1-\kappa) z, \tag{11}$$

where κ is a constant. Then, $E_i[\overline{a}] = E[\overline{a}|z, x_i] = E\left[\int_0^1 a_j(z)\, dj | z, x_i\right] = = E\left[\int_0^1 (\kappa x_j + (1-\kappa) z)\, dj | z, x_i\right] = \kappa \int_0^1 E[x_j|z, x_i]\, dj + (1-\kappa) z$, and $E[x_j|z, x_i] = \frac{\alpha z + \beta x_i}{\alpha + \beta}$, so that $E_i[\overline{a}] = \kappa \frac{\alpha z + \beta x_i}{\alpha + \beta} + (1-\kappa) z = \frac{\kappa\beta}{\alpha+\beta} x_i + \left(1 - \frac{\kappa\beta}{\alpha+\beta}\right)$. Then, individual i's optimal action follows from (10), so that $a_i(z, x_i) = (1-\lambda) \frac{\alpha z + \beta x_i}{\alpha + \beta} + \lambda \left[\frac{\kappa\beta}{\alpha+\beta} x_i + \left(1 - \frac{\kappa\beta}{\alpha+\beta}\right)\right]$ and

$$a_i(z, x_i) = \frac{\beta(\lambda\kappa + 1 - \lambda)}{\alpha + \beta} x_i + \left(1 - \frac{\beta(\lambda\kappa + 1 - \lambda)}{\alpha + \beta}\right) z. \tag{12}$$

Combining (11) and (12), we obtain $\kappa = \frac{\beta(\lambda\kappa + 1 - \lambda)}{\alpha + \beta} \Leftrightarrow \kappa = \frac{\beta(1-\lambda)}{\beta(1-\lambda) + \alpha}$. Hence, $a_i = \frac{\alpha z + \beta(1-\lambda) x_i}{\alpha + \beta(1-\lambda)}$, which confirms our initial guess.

4.2.1 Welfare implications

We are ready to answer a fundamental question. Is welfare increasing in the precision α of public information? The answer is: it depends. From the solution for a_i in Proposition 4, and the definitions of the signals z and x_i, we obtain $a_i = \theta + \frac{\sqrt{\alpha}\varepsilon + \sqrt{\beta}(1-\lambda)\varepsilon_i}{\alpha + \beta(1-\lambda)}$. For a given state θ, the expected welfare is equal to

$$\begin{aligned}
E[W(\mathbf{a},\theta)|\theta] &= E\left[-\int_0^1 (a_i-\theta)^2\, di|\theta\right] = -E\left[\int_0^1 \left(\frac{\sqrt{\alpha}\varepsilon+\sqrt{\beta}(1-\lambda)\varepsilon_i}{\alpha+\beta(1-\lambda)}\right)^2 di|\theta\right] \\
&= -\frac{\alpha+\beta(1-\lambda)^2}{[\alpha+\beta(1-\lambda)]^2}. \qquad (13)
\end{aligned}$$

By examining (13), we can assess the effects of increasing the precision of public information and private information. First, welfare is always increasing in the precision of the private signals. It is easy to check that $\frac{\partial E[W(\mathbf{a},\theta)|\theta]}{\partial\beta} > 0$.

Second, the effect of the precision of public information on welfare is ambiguous because $\frac{\partial E[W(\mathbf{a},\theta)|\theta]}{\partial\alpha} = \frac{\alpha-(2\lambda-1)(1-\lambda)\beta}{[\alpha+\beta(1-\lambda)]^3}$. Since $\alpha+\beta(1-\lambda)>0$, welfare is increasing in the precision of the public signal if and only if $\frac{\partial E[W(\mathbf{a},\theta)|\theta]}{\partial\alpha} \geq 0 \Leftrightarrow \alpha-(2\lambda-1)(1-\lambda)\beta \geq 0$, that is

$$\frac{\beta}{\alpha} \leq \frac{1}{(2\lambda-1)(1-\lambda)}. \qquad (14)$$

When $\lambda < 0.5$, the above condition is trivially satisfied since α and β are positive. But when $\lambda > 0.5$, there are parameter values for which the increased precision of public information is detrimental to welfare. When the coordination motive is very important (λ is large), better public information may be harmful.

It is instructive to compare these results with the case in which there is no Keynesian "beauty contest" component. In this last case, $\lambda = 0$ and from (13) we obtain $E[W(\mathbf{a},\theta)|\theta] = -\frac{1}{\alpha+\beta}$. In this case, better private and public information is always beneficial for welfare.

Svensson's Critique. The main result of Morris and Shin (2002) has often been interpreted as an anti-transparency result. Svensson (2006) challenges this view, arguing that only under very specific conditions more transparency can have a negative impact. He performs sensitivity analysis using Morris and Shin (2002)'s model, and concludes that condition (14) is very likely to be satisfied under reasonable assumptions. Hence, more transparency is generally good for social welfare.

5 Transparency of Information and Investment Complementarities

Angeletos and Pavan (2004), in their paper *Transparency of Information and Coordination Economies with Investment Complementarities*, explore how the precision of public and private information impacts on equilibrium allocations and social welfare. Unlike Morris and Shin (2002), they find that social welfare always increases with the precision of public information (interestingly, they find that the precision of private information has an ambiguous effect on welfare).

In this section, we return to an environment with strategic complementarities at the social level, that we have already analyzed in the bank run game and in Angeletos and Werning (2006). This environment is useful to analyze economies with production externalities, and models with macroeconomic complementarities.

Methodologically, Angeletos and Pavan (2004) introduce an elegant and tractable payoff function so as to obtain a unique equilibrium. They consider increasing investment costs, which prevents extreme actions by individuals and "bang-bang" solutions.

5.1 The Model

There is a continuum of individuals represented by the interval $[0,1]$. Individual i's payoff function is

$$u(k_i, A) = Ak_i - \frac{1}{2}k_i^2 \tag{15}$$

where $k_i \in \mathbb{R}$ represents individual investment. This assumption is important for uniqueness because, first, k_i a continuous variable and, second, there are large costs associated with undertaking extreme actions, like $k_i \to -\infty$ or $k_i \to +\infty$, and all individuals prefer "intermediate" levels on investment. Aggregate investment equals $K = \int_0^1 k_i di$.

There is complementarity at the social level because the individual return A is increasing in the aggregate level of investment K, as

$$A = (1-\lambda)\theta + \lambda K$$

where θ represents the exogenous return to investment (that is, the underlying fundamental), and parameter $\lambda \in \left[0, \frac{1}{2}\right)$ measures the degree of complementarity.

Social welfare is given by $W(\mathbf{k}, \theta) = \int_0^1 u(k_i, A)\, di$. Using expression (15) we obtain

$$W(\mathbf{k}, \theta) = AK - \frac{1}{2}\int_0^1 k_i^2 \, di,$$

where $\mathbf{k}$ represents the profile of investment for all individuals. Adding and subtracting

$\frac{1}{2}\int_0^1 K^2 - 2k_i K di$, yields

$$\begin{aligned} W(\mathbf{k},\theta) &= AK + \frac{1}{2}\int_0^1 \left(K^2 - 2k_i K\right) di - \frac{1}{2}\int_0^1 \left(k_i^2 - 2k_i K + K^2\right) di \\ &= AK - \frac{1}{2}K^2 - \frac{1}{2}var \end{aligned} \tag{16}$$

where $var = \int_0^1 (k_i - K)^2\, di$ measures the cross-sectional heterogeneity in investment. Unlike Morris and Shin (2002), the social planner cares about reducing heterogeneity in individual actions.

The structure of information is as in Morris and Shin (2002). The common prior about θ is uniform over $\mathbb{R}$, there is a public signal $z = \theta + \frac{1}{\sqrt{\alpha}}\varepsilon$ and a private signal $x_i = \theta + \frac{1}{\sqrt{\beta}}\varepsilon_i$ where ε and ε_i have the same properties as before. Under the current framework, we obtain the following result (similar to Proposition 4).

Proposition 5 *There is a unique equilibrium, given by* $k_i = \frac{\beta(1-\lambda)}{\alpha+\beta(1-\lambda)}x_i + \left(1 - \frac{\beta(1-\lambda)}{\alpha+\beta(1-\lambda)}\right)z$.

5.2 Welfare implications

Consider social welfare evaluated at equilibrium. Expression (16) can be rewritten as

$$W(\mathbf{k},\theta) = (1-\lambda)\,\theta K - (1-2\lambda)\,\frac{1}{2}K^2 - \frac{1}{2}var.$$

In the next result, we relate welfare with the precision of information.

Proposition 6 *Expected welfare conditional on the realization of the fundamental* θ *equals* $E\left[W(\mathbf{k},\theta)\,|\theta\right] = \frac{1}{2}\theta^2 - \frac{1}{2}\frac{(1-2\lambda)\alpha+\beta(1-\lambda)^2}{[\alpha+\beta(1-\lambda)]^2}$.

In contrast with Morris and Shin (2002), more precise public information necessarily increases welfare, because $\frac{\partial E[W(\theta,z)|\theta]}{\partial\alpha} = \frac{1}{2}\frac{(1-2\lambda)\alpha+\beta(1-\lambda)(1+2\lambda)}{[\alpha+\beta(1-\lambda)]^3} > 0$. More precise private information has an ambiguous effect on welfare.

The difference in the results stems from an important distinction in the two models. In Morris and Shin (2000), the social planner seeks to keep all agents' actions close to the fundamental θ, and does not care about reducing heterogeneity in individual actions - as shown in expression (9). Any attempts to align individual actions are socially wasteful. Yet, individuals value more effective coordination, because of the "beauty contest" component of their utility function. Individuals use public information to second-guess what others will do, and this creates negative externalities as individuals "overreact" to public information.

Angeletos and Pavan (2004) consider a setup in which the strategic complementarities are present at the social level, so that the social planner cares about coordinating individual investment choices - as shown in expression (16). Individuals use public information to align their actions, and more transparent public information allows more effective coordination, thereby increasing welfare.

6 Conclusion

We reviewed influential applications of global games to macroeconomics and finance. We collected in a single paper the most relevant contributions in the field, and we identified the main areas of debate in the literature. For a more advanced treatment of global games, see the contribution by Morris and Shin (2003) in which the authors examine in detail the theoretical underpinnings of global games.

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A Appendix

This document is an electronic appendix to the paper *A Primer on Global Games Applied to Macroeconomic and Finance* by José Jorge and Joana Rocha.

This appendix has been refereed with the text. However, no attempt has been made to impose a uniform editorial style on this appendix. This appendix contains proofs of some results in the paper.

A.1 Proof of Proposition 4

It remains to show the uniqueness of equilibrium. From condition (10),

$$\begin{aligned}
a_i &= (1-\lambda) E[\theta|z,x_i] + \lambda E[\overline{a}|z,x_i] \\
&= (1-\lambda) E[\theta|z,x_i] + \lambda E\left[\int_0^1 a_j \; dj|z,x_i\right] \\
&= (1-\lambda) E[\theta|z,x_i] + \lambda E\left[\int_0^1 (1-\lambda) E[\theta|z,x_j] + \lambda E[\overline{a}|z,x_j]\, dj|z,x_i\right] \\
&= (1-\lambda) E[\theta|z,x_i] + \lambda(1-\lambda)\int_0^1 E[E[\theta|z,x_j]|z,x_i]\, dj + \lambda^2\left[\int_0^1 E[E[\overline{a}|z,x_j]|z,x_i]\, dj\right] \\
&= (1-\lambda) E[\theta|z,x_i] + \lambda(1-\lambda) E[E[\theta|z,x_j]|z,x_i] + \lambda^2 E[E[\overline{a}|z,x_j]|z,x_i].
\end{aligned}$$

Iterating one more time,

$$\begin{aligned}
a_i &= (1-\lambda) E[\theta|z,x_i] + \lambda(1-\lambda) E[E[\theta|z,x_j]|z,x_i] + \lambda^2 E\left[E\left[\int_0^1 a_\iota d\iota|z,x_j\right]|z,x_i\right] \\
&= (1-\lambda) E[\theta|z,x_i] + \lambda(1-\lambda) E[E[\theta|z,x_j]|z,x_i] + \\
&\quad +\lambda^2 E\left[E\left[\int_0^1 (1-\lambda) E[\theta|z,x_\iota] + \lambda E[\overline{a}|z,x_\iota]\, d\iota|z,x_j\right]|z,x_i\right] \\
&= (1-\lambda) E[\theta|z,x_i] + \lambda(1-\lambda) E[E[\theta|z,x_j]|z,x_i] + \\
&\quad +\lambda^2(1-\lambda) E[E[E[\theta|z,x_\iota]|z,x_j]|z,x_i] + \lambda^3 E[E[E[\overline{a}|z,x_\iota]|z,x_j]|z,x_i].
\end{aligned}$$

Hence, from recursive substitution, we obtain

$$a_i = (1-\lambda) E[\theta|z,x_i] + \lambda(1-\lambda) E[E[\theta|z,x_j]|z,x_i] + \lambda^2(1-\lambda) E[E[E[\theta|z,x_\iota]|z,x_j]|z,x_i] + \ldots \tag{17}$$

with the last order term converging to zero as the number of terms converge to infinity. Morris and Shin (2002) prove a Lemma, in which they show the following pattern:

$$\begin{aligned}
E\left[\theta|z,x_i\right] &= \left[1-\frac{\beta}{\alpha+\beta}\right]z+\frac{\beta}{\alpha+\beta}x_i \\
E\left[E\left[\theta|z,x_j\right]|z,x_i\right] &= \left[1-\left(\frac{\beta}{\alpha+\beta}\right)^2\right]z+\left(\frac{\beta}{\alpha+\beta}\right)^2x_i \\
E\left[E\left[E\left[\theta|z,x_\iota\right]|z,x_j\right]|z,x_i\right] &= \left[1-\left(\frac{\beta}{\alpha+\beta}\right)^3\right]z+\left(\frac{\beta}{\alpha+\beta}\right)^3x_i
\end{aligned}$$

and so forth. These expressions allow to rewrite (17) as a series

$$a_i=(1+\lambda)\sum_{\tau=0}^{\infty}\lambda^\tau\left[1-\left(\frac{\beta}{\alpha+\beta}\right)^\tau\right]z+\left(\frac{\beta}{\alpha+\beta}\right)^\tau x_i$$

which converges to $a_i=\frac{\alpha z+\beta(1-\lambda)x_i}{\alpha+\beta(1-\lambda)}$.■

A.2 Proof of Proposition 5

We only show the existence of equilibrium. Each individual i wants to maximize its expected utility $E\left[u\left(k_i,A\right)|z,x_i\right]$. The first order condition with respect to k_i yields

$$k_i=E\left[A|z,x_i\right]=(1-\lambda)\,E\left[\theta|z,x_i\right]+\lambda E\left[K|z,x_i\right].$$

The expectation operator is a linear operator and, as result, k_i is a linear function of x_i and z. We guess that $k_i=\varrho_x x_i+\varrho_z z$, where ϱ_x and ϱ_z are constants to be determined in equilibrium. Then,

$$K=\int_0^1 k_i\;di=\int_0^1\left(\varrho_x x_i+\varrho_z z\right)di=\varrho_x\theta+\varrho_z z$$

and

$$\begin{aligned}
k_i &= (1-\lambda)\,E\left[\theta|z,x_i\right]+\lambda E\left[\varrho_x\theta+\varrho_z z|z,x_i\right] \\
&= (1-\lambda)\frac{\alpha z+\beta x_i}{\alpha+\beta}+\lambda\left(\varrho_x\frac{\alpha z+\beta x_i}{\alpha+\beta}+\varrho_z z\right) \\
&= \frac{\beta}{\alpha+\beta}\left(1-\lambda+\lambda\varrho_x\right)x_i+\left[\left(1-\lambda+\lambda\varrho_x\right)\frac{\alpha}{\alpha+\beta}+\lambda\varrho_z\right]z.
\end{aligned}$$

The initial guess is verified with $\varrho_x=\frac{\beta}{\alpha+\beta}\left(1-\lambda+\lambda\varrho_x\right)$ and $\varrho_z=\left(1-\lambda+\lambda\varrho_x\right)\frac{\alpha}{\alpha+\beta}+\lambda\varrho_z$, which implies $\varrho_x=\frac{\beta(1-\lambda)}{\alpha+\beta(1-\lambda)}$ and $\varrho_z=1-\varrho_x$. ■

A.3 Proof of Proposition 6

Since $W(\mathbf{k},\theta) = (1-\lambda)\theta K - (1-2\lambda)\frac{1}{2}K^2 - \frac{1}{2}var$, expected welfare conditional on the realization of the fundamental θ equals

$$E\left[W(\mathbf{k},\theta)|\theta\right] = (1-\lambda)\theta E\left[K|\theta\right] - (1-2\lambda)\frac{1}{2}E\left[K^2|\theta\right] - \frac{1}{2}Var\left[k_i|\theta\right], \tag{18}$$

where

$$\begin{aligned} Var\left[k_i|\theta\right] &= E\left[\int_0^1 (k_i-K)^2\, di|\theta\right] = E\left[\int_0^1 \left[\varrho_x x_i + \varrho_z z - (\varrho_x\theta + \varrho_z z)\right]^2 di|\theta\right] \\ &= \varrho_x^2 E\left[\int_0^1 (x_i-\theta)^2\, di|\theta\right] = \frac{\varrho_x^2}{\beta} \end{aligned}$$

is the equilibrium value of heterogeneity. Moreover,

$$E\left[K|\theta\right] = E\left[\int_0^1 k_i\ di|\theta\right] = E\left[\int_0^1 (\varrho_x x_i + \varrho_z z)\, di|\theta\right] = \theta$$

and, by the definition of variance,

$$E\left[K^2|\theta\right] = Var\left[K|\theta\right] + \left(E\left[K|\theta\right]\right)^2 = Var\left[K|\theta\right] + \theta^2$$

where

$$\begin{aligned} Var\left[K|\theta\right] &= E\left[\left(K - E\left[K\right]\right)^2|\theta\right] = E\left[(K-\theta)^2|\theta\right] = E\left[\left(\varrho_x\theta + (1-\varrho_x)z - \theta\right)^2|\theta\right] \\ &= E\left[(1-\varrho_x)^2(z-\theta)^2|\theta\right] = \frac{\varrho_z^2}{\alpha} \end{aligned}$$

is the equilibrium value of volatility. Replacing these results in expression (18) we obtain

$$\begin{aligned} E\left[W(\mathbf{k},\theta)|\theta\right] &= (1-\lambda)\theta^2 - (1-2\lambda)\frac{1}{2}\left(\frac{\varrho_z^2}{\alpha} + \theta^2\right) - \frac{1}{2}\frac{\varrho_x^2}{\beta} \\ &= \frac{1}{2}\theta^2 - \frac{1}{2}\left[(1-2\lambda)\frac{\varrho_z^2}{\alpha} + \frac{\varrho_x^2}{\beta}\right] = \frac{1}{2}\theta^2 - \frac{1}{2}\frac{(1-2\lambda)\alpha + \beta(1-\lambda)^2}{\left[\alpha + \beta(1-\lambda)\right]^2}. \end{aligned}$$

■

Financial Intermediation in Economies with Investment Complementarities*

Essay 2

July 2015

Abstract

When individual returns are increasing in the aggregate level of investment, decentralized individuals fail to internalize the positive externality of their investment on the return of others. This paper shows how financial intermediation mitigates this coordination failure for individuals with private information. When providing financial products with low risk, intermediaries induce individuals with unfavorable private information to invest more. The increase in investment generates positive externalities, thereby raising social welfare and making banks socially desirable.

Keywords: Banking, Macroeconomics, Incomplete Information, Coordination, Complementarities, Externalities

JEL Classification Codes: G21, E44, D82, C72, D62

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1 Introduction

Confidence and expectations are critical to determine equilibrium allocations in economies with production externalities since there is the possibility of coordination failure. Decentralized individuals do not internalize the positive externality of their investment on the return of others, thereby investing too little. Within this class of economies, how do financial intermediaries permit more effective coordination in the market? And how do intermediaries affect equilibrium allocations and social welfare?

To answer these questions, we consider an economy with production externalities in which individuals have private information about the underlying economic fundamentals. Our analysis builds on the model of Angeletos and Pavan (2004) in which the return to individual investment is increasing in the aggregate level of investment. Their framework has two main advantages. First, it is possible to compute social welfare explicitly and, second, results do not hinge on the volatility of the underlying economic fundamentals. We extend their model by adding financial intermediaries which transfer funds from a pool of investors to a pool of firms. Intermediaries are able to reduce the volatility of the underlying economic fundamentals of individual firms by monitoring them and, as a result, intermediaries are able to offer financial products which pay a relatively constant return across states.

The novel contribution of the paper is to show how financial intermediation alleviates the coordination failure which arises in economies with investment externalities. In a nutshell, the central argument of the paper is that intermediaries are able to create safe assets which entice investment by those individuals with unfavorable private information—a flight-to-quality effect by pessimistic individuals. Production externalities raise individual returns throughout the economy, further stimulating investment and increasing social welfare.

To fix ideas, consider the following example. A large number of investors are choosing how much to invest in a new sector. The profitability of the firms in this sector depends on an exogenous productivity parameter—which we call the economic fundamentals— and on aggregate investment. The investors thus have an incentive to align their choices, which makes individual investment overly sensitive to public information about the fundamentals.

In Section 2, we consider a market-based financial system in which individuals with homogenous expectations invest directly in firms, and show how coordination failures generate underinvestment. Better public information is beneficial for social welfare, since investors use public information to fine-tune their choices and take more efficient decisions.

In Section 3, we consider the existence of private information, which introduces heterogeneity in expectations about the underlying fundamentals. Heterogenous beliefs engender cross-sectional heterogeneity in investment choices, as pessimistic individuals (who received unfavorable private information) invest less than optimistic individuals.

We then consider the existence of financial intermediaries which reduce the uncertainty in the fundamentals, by alleviating the agency problems resulting from the relationship between investors and firms. One important problem arises if the manager of the firm must take some action to make proper use of the funds they have obtained from investors. For example, the manager may have the possibility to choose between two projects: one with high risk and private benefits and another with low risk and no private benefits. Investors cannot observe the manager's decisions, but the financial intermediary can observe the manager's actions by paying a monitoring cost. Hence, investors hire the intermediary to check what the manager is doing, and prevent him from choosing the riskiest project.[1] By monitoring the firm, financial intermediaries are able to transform risky investment projects into safe projects, thus enabling intermediaries to offer safe financial products to investors. Indeed, traditional banking activities transform risky investment in firms into safer financial assets, like time deposits.

We contrast the results between a financial system based exclusively on direct finance, and a financial system with coexistence between direct and intermediated finance. Pessimistic individuals prefer investing in safer financial products offered by financial intermediaries rather than investing directly in firms. Pessimistic individuals end up investing more than they would invest in a market-based financial system, thus raising aggregate investment. Individual returns increase as a result of investment complementarities, thereby inducing investment by optimistic investors. Aggregate investment and social welfare increase with coexistence between intermediated and direct finance.

Since it is possible to compute social welfare explicitly, we provide policy recommendations on how to mitigate coordination failures. Our results suggest stimulating financial intermediation when the degree of strategic complementarity between firms is large and there is substantial uncertainty regarding the fundamentals.

Our analysis has implications not only for economic policy but also for empirical work. Our model provides testable implications regarding the extent to which firms, industries, and regions can be expected to suffer from restrictions in intermediated finance or can be expected to benefit from government policies which boost indirect finance. In particular, changes in bank lending and credit policies will have the most impact where strategic complementarities are the most prevalent.

[1] We assume that only the intermediation sector has access to the monitoring technology, or it is efficient to have a bank as a *delegated monitor*. In Germany and Japan, banks have large equity stakes in large corporations and perform a very important corporate governance role in large corporations.

Review of the literature. Industries with production externalities are one example in which strategic complementarities play a prominent role. The individual firm's production function displays production externalities when the productivity of the individual firm increases with aggregate production. For instance, Cooper and John (1988) consider a model with technological complementarities among input suppliers to a shared production process of a public good, while Bryant (1983) shows that specialization and imperfect information lead to strategic complementarities among producers. In both frameworks, an increase in aggregate production will raise individual gains.

Another justification for the existence of production externalities is Alfred Marshall's concept of external scale economies. According to Marshall (1890), there are three sources of external scale economies at the firm level. First, there is the potential for more extensive interaction between suppliers and buyers, allowing for productivity gains resulting from vertical disintegration and supplier specialization. In a similar vein, Diamond's (1982) search model assumes that an increase in the number of potential trading partners makes trade easier, which in turn makes production more efficient. Second, there is the firm's ability to capture industry-specific knowledge and information spillovers which take place in related industries (as in Carvalho and Voigtländer 2014). Third, there are benefits from a larger pool of skilled labor associated with a stronger industry, and which favors the firm-worker matching process.

The endogenous growth literature has also provided several justifications for the existence of production externalities. According to Romer (1986) and Lucas (1988), capital includes both physical and human components, and two key assumptions generate technological complementarities. First, knowledge creation is a side effect of physical investment. A firm that increases its physical capital learns simultaneously how to produce more efficiently. Second, each firm's knowledge is a public good that any other firm can access at zero cost. Once discovered, a piece of knowledge spills over across the whole economy so that all firms can benefit from it. Alternatively, Barro (1990) shows that tax-financed government services are another possible source of production externalities. In this case, the government's choices determine the productivity in the economy.

External economies also play an important role in shaping the pattern of international trade, and are decisive in shaping the pattern of interregional trade. Researchers in international trade and economic geography have joined geographers and urban economists in investigating the relationship between production externalities and geographical agglomeration (see, for example, Krugman 1991a, 1991b). Below, we suggest using geographical agglomeration as a measure of production externalities.

A number of authors have embedded technological complementarities in general equilibrium models. Baxter and King (1991) and Benhabib and Farmer (1994) build dynamic stochastic general equilibrium models with technological complementarities and a representative agent, whereas Acemoglu (1993) considers a model with technological complementarities and heterogenous individuals.

Morris and Shin (2002) analyze an environment with strategic complementarities and heterogenous information. Since complementarities are present only at the private level, they find that more transparent public information can reduce welfare. Unlike Morris and Shin (2002), Angeletos and Pavan (2004) consider an economy in which complementarities are present at the social level, so that more precise public information necessarily increases welfare. Still, none of these authors consider changes in the volatility of the underlying fundamentals, as we do.

The finance literature has used bankruptcy as an instrument to identify channels for (negative) spillover effects among firms. Lang and Stulz (1992) and Ferris, Jayaraman, and Makhija (1997) document spillover effects of bankruptcy filings on investors of industry peers. Hertzel, Li, Officer, and Rodgers (2008) examine bankruptcy contagion effects along the supply chain of filing firms, while Boone and Ivanov (2012) define proximate non-filing firms as strategic alliance partners. Jorion and Zhang (2007) and Hertzel and Officer (2012) document bankruptcy contagion effects on industry capital providers. Addoum, Kumar, Le, Niessen-Ruenzi (2015) document that, following the bankruptcy of a geographically proximate firm, firms that are located geographically near the bankrupt firm reduce their investment expenditures. They document worse credit conditions for local firms if a local firm files for bankruptcy.

In a model with strategic complementarities and bank lending, Bebchuk and Goldstein (2011) show that firms are vulnerable to credit market freezes. Banks avoid lending to firms out of self-fulfilling fear (validated in equilibrium) that other banks would withhold loans to firms, thus causing their default. Like Bebchuk and Goldstein, we also suggest policies to mitigate the coordination problem, and point out a number of empirical implications. Still, there are important differences with our paper. First, we model explicitly the technology and the preferences in the economy. Second, we consider direct finance from households to operating firms and not just intermediated lending. Third, we take a broader view of strategic complementarities which enables us to make policy recommendations regarding business fluctuations, and not just extreme conditions such as credit market freezes. Fourth, and most importantly, our framework enables us to compute social welfare which in turn allows us to quantify the welfare implications of the policy measures.

A number of recent papers focus on policy issues when there are strategic complementarities. In a coordination model akin to ours, Sákoviks and Steiner (2012) identify the optimal policy of investment subsidies. Subsidies should be targeted at those firms (i) whose investment has relatively large spillover effects on the economy (as we suggest), and (ii) which are relatively insensitive to the investment of others themselves (as they consider unlike us heterogeneous strategic complementarities across firms). Philippon and Schnabl (2013) analyze government interventions to recapitalize a banking system which suffers from coordination problems and restricts lending to firms (due to debt overhang). The efficient recapitalization policy injects capital in the banking system, thus alleviating the coordination problems among banks and augmenting firms' investment.

2 The Model

There is a continuum of investors indexed by i and uniformly distributed in interval $[0,1]$. The utility of investor i equals

$$u_i = Ak_i - \frac{1}{2}k_i^2$$

where $k_i \in \mathbb{R}$ represents individual investment, A denotes the individual return to investment, and $k_i^2/2$ is the individual cost of investment. The aggregate level of investment is given by $K = \int_0^1 k_i di$. As in Benhabib and Farmer (1994), Acemoglu (1996), Romer (1996), and Angeletos and Pavan (2004), strategic complementarities are embodied in the return A, as the individual return is increasing in the aggregate level of investment. Formally,

$$A = (1-\lambda)\theta + \lambda K.$$

The individual return A depends on the underlying exogenous economic fundamentals θ and on the aggregate level of investment K, while $\lambda \in \left[0, \frac{1}{2}\right)$ parametrizes the degree of strategic complementarity. Finally, social welfare equals

$$W = \int_0^1 u_i di = AK - \frac{1}{2}\int_0^1 k_i^2 di = (1-\lambda)\theta K - (1-2\lambda)\frac{1}{2}K^2 - \frac{1}{2}\int_0^1 (k_i - K)^2 di.$$

As a result of strategic complementarities, social welfare depends both on the economic fundamentals and on aggregate investment. The term $\frac{1}{2}\int_0^1 (k_i - K)^2 di$ represents cross-sectional heterogeneity in investment decisions.

2.1 Market-based finance

Under market-based finance, k_i represents the direct investment of individual i in a representative firm. Individuals choose k_i to maximize their utility.

If θ were known, individuals would set $k_i = \theta$ and all investors would invest the same amount. Yet, the first-best prescribes setting a level of individual investment k_i larger than θ. Decentralized individuals do not internalize the positive externality of their investment on the return of others.[2]

We now examine the cases in which the underlying economic fundamentals θ are uncertain. The exogenous return θ is not known at the time the investment decisions are made. Unlike Angeletos and Pavan (2004), we assume that the underlying economic fundamentals θ are a normal random variable with mean $\overline{\theta}$ and variance $\frac{1}{\gamma}$. Investor i maximizes its expected utility $E_i[u_i]$, so that optimal individual investment is given by

$$k_i = (1-\lambda)E_i[\theta] + \lambda E_i[K].$$

Individual investment is an increasing linear function of the expected economic fundamentals and the expected aggregate investment.

Proposition 1 *The equilibrium exists, is unique and given by $k_i = \overline{\theta}$. Ex ante social welfare is given by $E[W] = \frac{1}{2}\overline{\theta}^2$.*

Individual investment is constant, so that the volatility in the economic fundamentals has no impact on k_i. Social welfare does not depend on the volatility of the economic fundamentals. Again, there is an underinvestment problem as the first-best level of individual investment is larger than $\overline{\theta}$.

[2] A Pigouvian corrective subsidy policy would implement the first-best allocation.

2.2 Market-based finance with public information

Consider that individuals receive an additional public signal z, such that

$$z = \theta + \frac{1}{\sqrt{\alpha}}\varepsilon$$

where ε is a standard normal random variable, independent of θ. The public signal z has precision α.

Proposition 2 *With public information, the equilibrium exists, is unique and given by* $k_i = \rho_1\overline{\theta}+\rho_2 z$ *with* $\rho_1 = \frac{\gamma}{\gamma+\alpha}$ *and* $\rho_2 = \frac{\alpha}{\gamma+\alpha}$*. Ex ante social welfare is given by* $E[W] = \frac{1}{2}\overline{\theta}^2+\frac{\alpha}{\gamma(\gamma+\alpha)}$*.*

The equilibrium investment k_i is a weighted average between the mean of the economic fundamentals $\overline{\theta}$ and the public signal z, with the weights depending on the variance of the fundamentals and on the precision of the public signal. All investors invest the same amount k_i, which varies with the public signal z.

It is efficient to set a high level of investment when the fundamentals are good and productivity is high. Increasing the precision of public information raises expected welfare, since the public signal z provides additional information about the fundamentals θ, thereby allowing investors to fine-tune their investment k_i to the exogenous return θ.

Such a fine-tuning effect provides a justification for promoting and regulating the disclosure of public information. It calls for stricter requirements regarding the disclosure of information by publicly traded companies, and demands incentives for the certification role by auditors or credit rating agencies.[3] It also entails increased transparency through disclosures from governments and other official institutions such as central banks.

Still, producing public information is not profitable. Financial intermediaries have no incentives to provide public information, if collecting information is costly. Intermediaries would be able to offer financial products identical to the ones already available to financial markets, but with lower return (since intermediaries would have to bear a cost to collect information and therefore charge fees to depositors). There is a free-rider problem, since everybody benefits from public information. For the rest of the paper, we do not consider the existence of a public signal z.

[3] A number of studies also suggests that bank loans provide public information to the market about the financial health of the firm (see, for example, James 1987). This is an additional channel through which financial intermediation is likely to have a positive effect on welfare.

3 Private information

We assume that each individual receives an additional piece of private information. Such information introduces heterogeneity in expectations about the fundamentals θ and may be understood as heterogeneity in the reading and interpretation of available information. With heterogenous beliefs about θ, there is cross-sectional heterogeneity in investment decisions, with optimistic individuals investing more than pessimistic individuals.

In Section 3.1 we investigate the case of market-based finance. Then, in Section 3.2 we consider the existence of a representative financial intermediary. The intermediary collects funds from individuals and invests these funds in firms. Monitoring enables the intermediary to offer an asset with less risk and lower expected return than direct investment in firms. Individuals can choose to invest directly in the firm, or they can choose to invest their funds through the financial intermediary. Pessimistic investors choose the safest option among the two investment alternatives, while optimistic investors choose market-based finance as it increases their potential gains.

In Section 3.3, we contrast the outcome in a market-based financial system with the outcome in a financial system with coexistence between intermediated and direct finance. Coexistence raises aggregate investment, thus increasing social welfare as a result of technological complementarities.

3.1 Market-based finance

There is a continuum of small firms financed directly by investors. Recall that the underlying economic fundamentals θ follow a normal random variable with mean $\overline{\theta}$ and variance $\frac{1}{\gamma}$. Consider that each investor receives a private signal

$$x_i = \theta + \frac{1}{\sqrt{\beta}}\varepsilon_i$$

where ε_i is standard normal, independent across investors and independent of θ, and β parametrizes the precision of private information.

Proposition 3 *With private information, equilibrium exists, is unique and given by* $k_i = \rho_3\overline{\theta} + \rho_4 x_i$ *with* $\rho_3 = \frac{\gamma}{\gamma+(1-\lambda)\beta}$ *and* $\rho_4 = \frac{(1-\lambda)\beta}{\gamma+(1-\lambda)\beta}$. *Ex ante social welfare is given by* $E[W] = (1-\lambda)\left[\overline{\theta}^2 + \frac{\rho_4}{\gamma}\right] - (1-2\lambda)\frac{1}{2}\left[\overline{\theta}^2 + \frac{\rho_4^2}{\gamma}\right] - \frac{1}{2}\frac{\rho_4^2}{\beta}$.

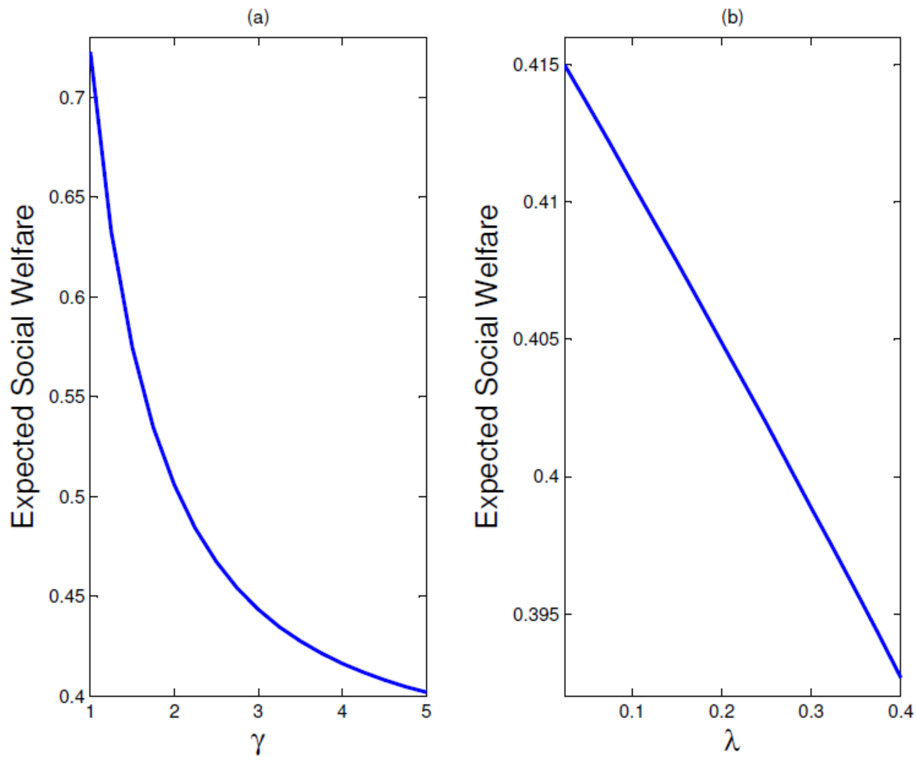

Figure 1: Expected social welfare with market-based finance as a function of γ and λ. Other parameters in this example: $\overline{\theta} = 0.85$ and $\beta = 4$; in (a) $\lambda = 0.25$ and in (b) $\gamma = 5$.

The functional form of equilibrium investment k_i is similar to the case with public information. Still, the weights of the two pieces of information in function k_i depend on the degree of strategic complementarity λ. If $\lambda = 0$, the two pieces of information would be given weights that are proportional to their precision (e.g., the private signal x_i would be given a weight equal to $\frac{\beta}{\gamma+\beta}$). The weights in the equilibrium strategy k_i deviate from these, so that the private signal is given relatively less weight. This property reflects the coordination motive arising from strategic complementarity in the actions of investors. It reflects the disproportionate influence of the public information embedded in the economic fundamentals, which individuals use to align their investment decisions.

What effects do the precision γ and the degree of strategic complementarity λ have on welfare? Expected welfare decreases with the precision of the fundamentals as individuals decrease the weight given to the private signal x_i and reduce the fine-tuning effect described above. The derivative of the expected social welfare $E[W]$ with respect to γ is negative, and this effect is illustrated in Figure 1(a) for specific values of $\overline{\theta}, \beta$, and λ.

Investors reduce the weight placed on private information as strategic complementarities increase, thus reducing the social benefit of the fine-tuning effect. Figure 1(b) depicts a numerical example showing a negative relationship between expected welfare and the degree of strategic complementarity λ. We performed a set of numerical simulations using grids for parameters β and γ to investigate if the results were sensitive to the combination of these parameters, and we verified that results were robust to all settings.[4]

3.2 Coexistence between intermediated and market-based finance

We want to analyze now if a financial intermediary that comes between the investors and the firms can make it possible to increase investment. The main objective is to show that intermediaries raise individual investment by pessimistic investors, thus improving aggregate productivity as a result of strategic complementarities.

Financial intermediation as delegated monitoring. We employ a standard model of investment with moral hazard. There is a continuum of firms, each managed by an entrepreneur. Each entrepreneur has the possibility to choose between two projects:

- The risky project offers pledgeable income A plus a private benefit to the manager of the firm.
- The safe project offers pledgeable income and no private benefit. The pledgeable income equals $(1-\lambda)\widehat{\theta}+\lambda K$ with
$$\widehat{\theta}=\overline{\theta}+\sqrt{\frac{\gamma}{\widehat{\gamma}}}(\theta-\overline{\theta})$$
where the random variable θ is a mean-preserving spread of $\widehat{\theta}$, so that $\sqrt{\frac{\gamma}{\widehat{\gamma}}}<1$ with $1/\widehat{\gamma}$ being the variance of $\widehat{\theta}$. Aggregate investment is equal to $\overline{K}$, and includes investment through the financial intermediary and direct investment in firms.

The choice of project by the entrepreneur is not observable (and therefore not contractible). Firms are perfectly competitive and entrepreneurs have limited liability. As a result, outside investors appropriate the pledgeable income of the project, and entrepreneurs keep the non-pledgeable part. It follows that entrepreneurs would rather implement the risky project.

Consider the existence of a representative financial intermediary. Some investors deposit their funds with the intermediary, which invests these funds in a pool of firms. Each firm is

[4] Numerical simulations for the paper may be found in the webpage http://www.fep.up.pt/docentes/jjorge/, under the tab "Research".

managed by an entrepreneur and financed either by the intermediary or directly by investors. As in Holmstrom and Tirole (1997), financial intermediaries monitor the firm at a cost $m > 0$ and prevent entrepreneurs from appropriating the private benefit of the risky project.[5] Entrepreneurs would rather obtain finance directly from uniformed investors, thus avoiding monitoring and implementing the risky project. With monitoring, they become indifferent between both projects—for simplicity, we assume that entrepreneurs choose the safe project. Only those entrepreneurs unable to obtain direct finance will seek informed capital from a financial intermediary.[6]

For simplicity, there are no conflicts of interest between the intermediary and its depositors, so that the intermediary will monitor the firms.[7] Intermediaries offer a financial product to individual investors with return equal to

$$\widehat{A} = (1-\lambda)\widehat{\theta} + \lambda\overline{K} - m$$

and individuals investing through a financial intermediary have utility

$$\widehat{u}_i = \widehat{A}\widehat{k}_i - \frac{1}{2}\widehat{k}_i^2$$

where $\widehat{k}_i$ represents the individual investment in the intermediary. Individuals who invest directly in firms benefit from an individual return

$$A = (1-\lambda)\theta + \lambda\overline{K}$$

[5]Monitoring encompasses (*i*) sreening projects when there is adverse selection as in Broecker (1990), (*ii*) preventing moral hazard in the implementation of investment projects as in Holmstrom and Tirole (1997), and (*iii*) auditing those projects who fail to meet contractual obligations, as in Townsend (1979) and Gale and Hellwig (1985). These monitoring activities can be performed by specialized firms which produce information such as rating agencies, auditors, or other financial analysts. Yet, the delegated monitoring theory of financial intermediation (see, for example, Diamond 1984 or Holmstrom and Tirole 1997) suggests that banks have a comparative advantage in these monitoring activities. Having a bank as a delegated monitor is advantageous when there are scale economies in monitoring, indivisibilities in investment projects, or low costs of delegation.

The literature often assumes that monitoring creates new investment opportunities which dominate available opportunities in the sense of first-order stochastic dominance. We require less, as we assume that the return on monitored projects second-order stochastically dominates the return on non-monitored projects.

[6]Alternatively, one could consider that the financial intermediary screens firms and issues securities which transfer risk from pessimistic to optimistic investors—in the spirit of Coval and Thakor (2005). Moral hazard or adverse selection are not indispensable assumptions. Rather, the only indispensable assumption is that the intermediary is able to reduce the risk of investment and therefore reduce the risk of the financial products being offered to their clients. The ability to offer products with little risk is a mild assumption, and a common result in the financial intermediation literature. For example, Allen and Gale (1997) document that financial intermediaries build up capital so as to offer an intertemporal smoothing of risk.

[7]The bank may be tempted not to monitor and appropriate m. The bank has no incentives to do this, since depositors would learn from the distribution of returns that the intermediary was not monitoring and would pick other investment alternatives or not invest at all.

and have utility

$$\widetilde{u}_i = A\widetilde{k}_i - \frac{1}{2}\widetilde{k}_i^2$$

where $\widetilde{k}_i$ represents direct individual investment in firms. Individuals compare the expected return obtained in both investment alternatives, and invest exclusively in the alternative which yields the highest expected return.

Since θ is a mean-preserving spread of $\widehat{\theta}$, then $E_i[A] > E_i[\widehat{A}]$ for sufficiently high x_i and $E_i[A] < E_i[\widehat{A}]$ for sufficiently low x_i. There is a threshold $\overline{x}$ for the private signal at which individuals are indifferent between investing directly in the firms or via an intermediary. Investors with a private signal x_i above the indifference threshold $\overline{x}$ prefer to invest directly in firms, whereas there is a group of individuals with signals below $\overline{x}$ who would rather invest in the intermediary. For this group of investors there is a flight-to-quality effect, but individuals with high private signals are not willing to invest in a financial product with low risk as it limits the upside potential.

We call *optimistic* to those investors who receive a private signal $x_i > \overline{x}$, and *pessimistic* to those investors who receive a private signal x_i below $\overline{x}$ and find optimal to invest through the financial intermediary. Only pessimistic investors are willing to switch from direct to intermediated finance.

If all pessimistic individuals invested through the financial intermediary, then the mass of investors in the intermediary would be variable. In this setting, the equilibrium investment decisions $\widetilde{k}_i$ and $\widehat{k}_i$ are not necessarily linear, in which case there is no analytical solution to the model.

In order to keep the analysis tractable enough to investigate the role of financial intermediation, we measure the marginal effect of adding a small financial intermediation system to the market equilibrium. To this end, we assume an economy or sector where financial intermediation is incipient. To put it more formally, we consider the limiting case when the financial intermediation sector is very small and only a small fraction of investors $\xi > 0$ can invest through the representative financial intermediary, and these investors cannot invest infinite amounts in the intermediary.

For those investors who choose direct finance, individual investment equals

$$\widetilde{k}_i = E_i[(1-\lambda)\theta + \lambda\overline{K}]$$

while investment for those investors who choose intermediated finance equals

$$\widehat{k}_i = E_i[(1-\lambda)\widehat{\theta} + \lambda\overline{K}] - m.$$

Aggregate investment equals

$$\overline{K} = \int\limits_{[0,1]\setminus B} \widetilde{k}_i di + \int\limits_{B} \widehat{k}_i di$$

where B is the set of investors who invest through the financial intermediary. The next result shows that there is a unique equilibrium with coexistence between direct and intermediated finance when the financial intermediation sector is sufficiently small and monitoring costs are not too high.

Proposition 4 *With private information, for sufficiently small values of ξ and m, there is a unique equilibrium in which financial intermediaries coexist with market-based finance. Equilibrium is given by $\widetilde{k}_i = \rho_5\overline{\theta} + \rho_6 x_i + \rho_7 m + \rho_8$ and $\widehat{k}_i = \rho_9\overline{\theta} + \rho_{10} x_i + \rho_{11} m + \rho_{12}$, with $\rho_5 \to \rho_3, \rho_6 \to \rho_4,\ \rho_7 \to 0, \rho_8 \to 0, \rho_9 \to \lambda\left(1 + \frac{\rho_4}{1-\lambda}\right)\frac{\gamma}{\gamma+\beta} + (1-\lambda)\frac{\widehat{\gamma}}{\widehat{\gamma}+\beta}, \rho_{10} \to \frac{(1-\lambda)\beta}{\widehat{\gamma}+\beta} + \frac{\lambda(1-\lambda)\beta^2}{[(\gamma+\beta)-\lambda\beta]\gamma+\beta}, \rho_{11} \to -1$ and $\rho_{12} \to 0$ as ξ converges to 0.*

In the proof of Proposition 4 we compute the marginal effect of introducing financial intermediaries in the market-based economy described in Section 3.1. As in the market-based economy, individual investment depends on $\overline{\theta}$ and x_i. Regarding the investment of those individuals who choose direct finance, the weights ρ_5 and ρ_6 are near the values obtained in Proposition 3. As for the investment decisions of those individuals who invest through the financial intermediary, the weights ρ_9 and ρ_{10} now depend on the variance of $\widehat{\theta}$.

Individual investment decisions also depend on the monitoring cost m. Lower monitoring costs raise the individual investment from those individuals who invest through the financial intermediary. Productivity increases as a result of investment complementarities, thereby enticing individuals who invest directly in firms to raise their investment. As a result, the coefficient ρ_7 converges to zero from below.

3.3 Contrasting a financial system based exclusively on market-based finance with a financial system with coexistence

In this section we compare the equilibrium in Proposition 3 with the equilibrium in Proposition 4, and show that coexistence raises aggregate investment and social welfare.

In the next result, we compare individual and aggregate investment in a financial system based exclusively on marked-based finance (obtained in Proposition 3) with individual and aggregate investment in a financial system with coexistence (obtained in Proposition 4).

Proposition 5 *In equilibrium with private information and for sufficiently small values of* ξ *and* m, $\widetilde{k}_i > k_i$ *for* $i \in [0,1] \setminus B$, $\widehat{k}_i > k_i$ *for* $i \in B$, *and* $\overline{K} > K$.

The value invested by those individuals who choose the financial intermediary is higher than the value they would invest if they chose direct finance. As a result of investment complementarities, all individuals invest more in the case of coexistence, thus increasing the level of aggregate investment.

The social welfare with coexistence equals the sum of investors' welfare, the financial intermediary's profit, firms' profits and managers' private benefits. We assume that perfectly competitive capital markets drive the profits of the representative financial intermediary and of firms to zero. With private benefits arbitrarily small, social welfare converges to investors' welfare. The next proposition compares welfare in a financial system based exclusively on market-based finance with welfare in a financial system with coexistence.

Proposition 6 *In equilibrium with private information and for sufficiently small values of* ξ *and* m, *ex ante social welfare increases with coexistence.*

Financial intermediation is socially desirable. Decentralized individuals do not internalize the positive externality of their investment on the return of others. Financial intermediaries raise individual investment, thus generating positive externalities and raising social welfare. Although there is a monitoring cost associated to investment through the intermediary, the effect of strategic complementarities dominates.

The impact of financial intermediation on social welfare is less relevant for less volatile fundamentals. In this case, individuals place little weight on private information, so that there is little dispersion of individual investment (individuals set their investment close to $\overline{\theta}$) and financial intermediaries have little impact on investment decisions. Figure 2(a) depicts a numerical example showing the percentage increase in expected welfare as the economy shifts from a market-based system to a financial system with coexistence between intermediaries and markets, as a function of γ when the ratio $\frac{\widehat{\gamma}}{\gamma}$ is constant. The figure suggests that financial intermediaries become more relevant as the volatility of fundamentals increases.

Financial intermediation is relevant if and only if there are strategic complementarities. Without strategic complementarities, raising individual investment does not increase individual return and welfare. The numerical example depicted in Figure 2(b) suggests that financial intermediaries add more welfare as the degree of strategic complementarity increases.

We performed numerical simulations for various combinations of parameters $\xi, \overline{\theta}, \beta, m, \lambda, \widehat{\gamma}$, and γ, and obtained qualitative results identical the ones plotted in Figure 2.

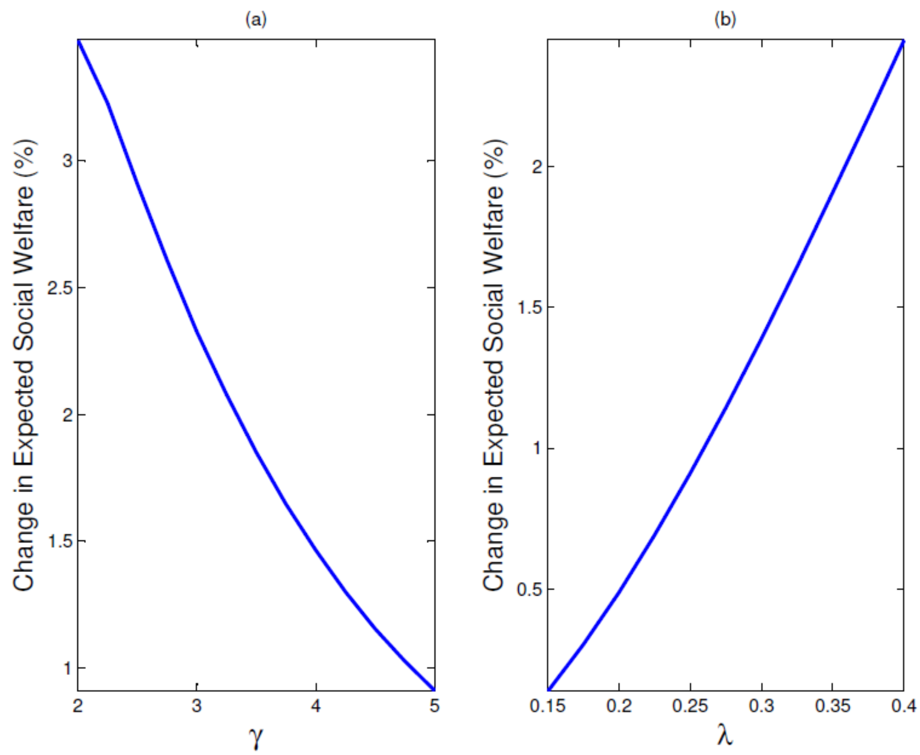

Figure 2: Change in expected social welfare from markets to coexistence when $\xi = 0.005\%$, as a function of γ and λ. Other parameters in this example: $\overline{\theta} = 0.85, \beta = 4, m = 10^{-5}, \widehat{\gamma} = 1.4\gamma$; in (a) $\lambda = 0.25$ and in (b) $\gamma = 5$.

3.4 Policy recommendations

The main contribution of our paper so far has been to show the possibility that financial intermediation adds to social welfare. Our results suggest that the size of the change in social welfare depends on the precision of the fundamentals γ and the degree of strategic complementarity λ, so that we shall now specify in which cases financial intermediation should be encouraged.

First, the welfare effect of financial intermediation increases when fundamentals are more volatile, thus suggesting that authorities should fine-tune their policies so as to incite intermediated finance to those industries which experience high uncertainty as a result of technological or regulatory shocks. In particular, credit lines targeted towards these industries would enhance investment and productivity. By the same token, those geographical areas which undergo periods of economic instability would benefit from favorable credit conditions.

At the aggregate level, our model prescribes raising the aggregate provision of intermediated finance in periods of macroeconomic uncertainty. For example, policy actions which influence the supply of bank credit will have an impact on investment if bank borrowers have no close substitutes to bank credit. Easy bank credit in periods of aggregate uncertainty would encourage intermediated loans so as to support firms' access to credit, fostering aggregate investment and returns, and raising aggregate welfare.

Second, intermediated finance is most useful in those industries with a substantial degree of strategic complementarity. Our results suggest that policy makers should promote specialized lending to those firms which benefit from external scale economies. Our model advises against subsidizing industries which do not benefit from strategic complementarities.

The equilibrium analysis in this section also provides a framework for analyzing and comparing specific government policies intended to promote lending.

(i) The infusion of capital into the banking system would raise the amount of intermediated funds, thus raising welfare. In our model, this would be equivalent to increasing the size of the representative financial intermediary.

(ii) Direct lending to firms would increase investment and welfare. In our model this would be equivalent to increasing the level of aggregate capital K. As in Bebchuk and Goldstein (2011), direct lending suffers from a disadvantage, as the government does not have the ability to monitor firms.

(iii) Government guarantees which provide funds to operating firms when they have low returns would increase individual investment and social welfare. These guarantees enable firms to offer stable returns to investors without wasting so much government resources as direct lending. Public guarantees would benefit from the advantages of monitoring, if these guarantees were channeled through specialized financial intermediaries.

3.5 Empirical implications

In addition to providing a framework for analyzing and evaluating government-supported mechanisms, our analysis also has substantial implications for empirical investigation.

First, Figure 2(b) suggests that financial intermediation is more important when strategic complementarities are most prevalent, so that a contraction in intermediated finance should have different impact across industries and geographical areas. A sharp test of our model would compare the impact of shocks on bank credit across industries and geographical areas with various degrees of strategic complementarity. One would expect industry clusters and regions with intense complementarities to be more sensitive to credit rationing. Bebchuk and Goldstein (2011) also suggest that sectors and regions with large strategic complementarities are more vulnerable to credit freezes.[8]

Second, policies which stimulate the supply of intermediated finance should have more impact on those industries and geographical areas where strategic complementarities are most prevalent. To the extent that bank lending depends on central banks' actions, monetary policy should have a differential impact across regions and across industries. Using data for the US, Carlino and DeFina (1998, 1999) document that monetary policy has a differential impact across regions, and some sectors of the economy, such as manufacturing, are more sensitive to monetary policy shocks than other sectors, such as services and retail. Yet, it remains to be shown that industry clusters and regions with intense complementarities are more sensitive to monetary policy shocks.

Third, agglomeration is widely recognized as a source and result of external scale economies. Jorge and Rocha (2016) use geographic concentration as a proxy for strategic complementarities, and document higher sensitivity to bank credit shocks among firms in industries with strong strategic complementarities.

Finally, our results suggest that supporting financial intermediation is likely to raise welfare significantly, and some of the responses to recent crises seem to conform to this belief. Using data for the Japanese banking crisis, Giannetti and Simonov (2013) show how bank bailouts had a positive effect on operating firms. Tong and Wei (2011) analyze 192 interventions for

[8] Still, Bebchuk and Goldstein (2011) highlight a different channel. They suggest that banks may refuse to lend to firms in sectors which benefit from strategic complementarities.

15 countries from September 2008 to July 2010, and show that unconventional monetary interventions aimed at inducing banks to be more willing to lend had a positive effect on non-financial firms.

Overall, our model highlights the need for and the value of empirical research which identifies the role of strategic complementarities in the relationship between financial intermediation and welfare.

4 Conclusion

We offer a stylized view of financial intermediation—our intermediary is rather similar to an institution which monitors and holds equity positions in firms—but one that is adequate for our purposes and is consistent with the results in the literature on financial intermediation.

We examine the welfare effects of introducing financial intermediaries in economies with investment complementarities. Decentralized individuals do not internalize the positive externality of their investment on the return of others, thereby investing too little. By monitoring firms and offering low risk financial products, intermediaries induce pessimistic individuals to invest more. Increased investment raises returns due to strategic complementarities, thus inducing more investment across the economy. In this way, intermediaries help to overcome the coordination failure among decentralized individuals, thus raising social welfare and making financial intermediation socially desirable.

Three extensions to the model may provide additional insights that have not been captured in the paper. First, we have assumed that financial intermediaries are special because they possess a monitoring technology which enables them to offer safe securities. Instead, one could consider that intermediaries have access to the same technology as other investors and assume that intermediaries use their capital to hedge the risk in the underlying economic fundamentals. Capital enables intermediaries to offer safe financial products by averaging risks across states. This involves depleting capital if the returns to financial intermediaries' assets are low, and accumulating gains if returns are high. Intermediaries can thus offer financial products which pay a relatively constant amount across states. Allen and Gale (1997) use a multiperiod model to describe how intermediaries build up their capital. They suggest that intermediaries can perform intertemporal smoothing in individual welfare, by averaging risks over time. This entails intermediaries building up reserves of safe assets when the returns on intermediaries' assets are high, and reducing them when returns are low.

Second, we have restricted the set of contracts available to investors. Investors and financial intermediaries have an equity stake in the firm and appropriate the whole surplus. Our *qualitative* results carry over to a less restrictive set of contracts, as long as financial intermediaries offer contracts with low risk. Still, enlarging the set of available contracts opens the debate on the *quantitative* significance of our results. Since the most common forms of intermediated finance—bank credit and bank deposits—have less risk than equity and reinforce the risk absorption by financial intermediaries, the existence of credit and deposit contracts is likely to strengthen the effects described in the paper.

Finally, we have performed our analysis for the particular case in which the financial intermediation system is small. In a model with strategic complementarities, Corsetti, Dasgupta, Morris and Shin (2004) show that a large player exercises a disproportionate influence on the behavior of small players. Extending our analysis to the case in which the intermediation system is large would allow for a more complete policy analysis.

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A Appendix

A.1 Proof of Proposition 1

We guess $k_i = \overline{\theta}$, so that $K = \int_0^1 k_i di = \int_0^1 \overline{\theta} di = \overline{\theta}$. Hence, $k_i = (1-\lambda)E_i[\theta] + \lambda E_i[K] = (1-\lambda)\overline{\theta} + \lambda\overline{\theta} = \overline{\theta}$ and the initial guess is verified. Equilibrium is unique as in Angeletos and Pavan (2004).

Expected social welfare is given by

$$E[W] = (1-\lambda)E[\theta K] - (1-2\lambda)\frac{1}{2}E\left[K^2\right] - \frac{1}{2}E\left[\int_0^1 (k_i - K)^2 di\right] = \frac{1}{2}\overline{\theta}^2$$

as $\int_0^1 (k_i - K)^2 di = 0$.■

A.2 Proof of Proposition 2

We guess $k_i = \rho_1\overline{\theta} + \rho_2 z$, so that $K = \int_0^1 k_i di = \int_0^1 (\rho_1\overline{\theta} + \rho_2 z)di = \rho_1\overline{\theta} + \rho_2 z$. Hence,

$$\begin{aligned}
k_i &= (1-\lambda)E_i[\theta] + \lambda E_i[K] = (1-\lambda)\frac{\gamma\overline{\theta} + \alpha z}{\gamma + \alpha} + \lambda E_i[\rho_1\overline{\theta} + \rho_2 z] = \\
&= (1-\lambda)\frac{\gamma\overline{\theta} + \alpha z}{\gamma + \alpha} + \lambda(\rho_1\overline{\theta} + \rho_2 z) \\
&= \left((1-\lambda)\frac{\gamma}{\gamma+\alpha} + \lambda\rho_1\right)\overline{\theta} + \left((1-\lambda)\frac{\alpha}{\gamma+\alpha} + \lambda\rho_2\right) z
\end{aligned}$$

and the initial guess is verified with

$$\begin{aligned}
\rho_1 &= (1-\lambda)\frac{\gamma}{\gamma+\alpha} + \lambda\rho_1 \Leftrightarrow \rho_1 = \frac{\gamma}{\gamma+\alpha} \\
\rho_2 &= (1-\lambda)\frac{\alpha}{\gamma+\alpha} + \lambda\rho_2 \Leftrightarrow \rho_2 = \frac{\alpha}{\gamma+\alpha}.
\end{aligned}$$

Expected social welfare is given by

$$\begin{aligned}
E[W] &= (1-\lambda)E[\theta K] - (1-2\lambda)\frac{1}{2}E\left[K^2\right] - \frac{1}{2}E\left[\int_0^1 (k_i - K)^2 d_i\right] = \\
&= (1-\lambda)E_i\left(\theta\frac{\gamma\overline{\theta} + \alpha z}{\gamma+\alpha}\right) - (1-2\lambda)\frac{1}{2}\left(Var[K] + (E[K])^2\right) = \\
&= (1-\lambda)\left(\overline{\theta}^2 + \frac{\alpha}{\gamma(\gamma+\alpha)}\right) - (1-2\lambda)\frac{1}{2}\left(\frac{\alpha}{\gamma(\gamma+\alpha)} + \overline{\theta}^2\right) \\
&= \frac{1}{2}\overline{\theta}^2 + \frac{\alpha}{\gamma(\gamma+\alpha)}.■
\end{aligned}$$

A.3 Proof of Proposition 3

We guess $k_i = \rho_3\overline{\theta} + \rho_4 x_i$, so that $K = \int_0^1 k_i di = \int_0^1 (\rho_3\overline{\theta} + \rho_4 x_i) d_i = \rho_3\overline{\theta} + \rho_4\theta$. Hence,

$$\begin{aligned}
k_i &= (1-\lambda)E_i[\theta] + \lambda E_i[K] = (1-\lambda)\frac{\gamma\overline{\theta} + \beta x_i}{\gamma+\beta} + \lambda E_i[\rho_3\overline{\theta} + \rho_4\theta] \\
&= (1-\lambda)\frac{\gamma\overline{\theta} + \beta x_i}{\gamma+\beta} + \lambda(\rho_3\overline{\theta} + \rho_4\frac{\gamma\overline{\theta} + \beta x_i}{\gamma+\beta}) \\
&= \left((1-\lambda)\frac{\gamma}{\gamma+\beta} + \lambda\rho_3 + \lambda\rho_4\frac{\gamma}{\gamma+\beta}\right)\overline{\theta} + \left((1-\lambda)\frac{\beta}{\gamma+\beta} + \lambda\rho_4\frac{\beta}{\gamma+\beta}\right)x_i
\end{aligned}$$

and the initial guess is verified with

$$\begin{aligned}
\rho_3 &= (1-\lambda)\frac{\gamma}{\gamma+\beta} + \lambda\rho_3 + \lambda\rho_4\frac{\gamma}{\gamma+\beta} \Leftrightarrow \rho_3 = \frac{\gamma}{\gamma+(1-\lambda)\beta} \\
\rho_4 &= (1-\lambda)\frac{\beta}{\gamma+\beta} + \lambda\rho_4\frac{\beta}{\gamma+\beta} \Leftrightarrow \rho_4 = \frac{(1-\lambda)\beta}{\gamma+(1-\lambda)\beta}
\end{aligned}$$

Expected social welfare is given by

$$\begin{aligned}
E[W] &= (1-\lambda)E[\theta K] - (1-2\lambda)\frac{1}{2}E\left[K^2\right] - \frac{1}{2}E\left[\int_0^1 (k_i - K)^2 di\right] \\
&= (1-\lambda)E[\theta K] - (1-2\lambda)\frac{1}{2}\left(Var[K] + (E[K])^2\right) - \frac{1}{2}E\left[\int_0^1 (k_i-K)^2 d_i\right] \\
&= (1-\lambda)\left(\overline{\theta}^2 + \frac{(1-\lambda)\beta}{\gamma(\gamma+(1-\lambda)\beta)}\right) \\
&\qquad - (1-2\lambda)\frac{1}{2}\left(\overline{\theta}^2 + \frac{((1-\lambda)\beta)^2}{\gamma(\gamma+(1-\lambda)\beta)^2}\right) - \frac{1}{2}\frac{((1-\lambda)\beta)^2}{\beta(\gamma+(1-\lambda)\beta)^2} \\
&= (1-\lambda)\left(\overline{\theta}^2 + \frac{\rho_4}{\gamma}\right) - (1-2\lambda)\frac{1}{2}\left(\overline{\theta}^2 + \frac{\rho_4^2}{\gamma}\right) - \frac{1}{2}\frac{\rho_4^2}{\beta}. \blacksquare
\end{aligned}$$

A.4 Proof of Proposition 4

We assume that investors cannot borrow or lend infinite amounts from the financial intermediary, that is $\widehat{k}_i \in [\underline{k}, \overline{k}]$ with $-\infty < \underline{k} < \overline{k} < +\infty$.

Lemma 1 *There is $\overline{x}$ and $\overline{\overline{x}}$ such that investors who receive a private signal $x_i \in [\overline{\overline{x}}, \overline{x}]$ find optimal to invest through the financial intermediary.*

Proof. The maximization problem of an investor who chooses financial intermediation is given by

$$\max_{\widehat{k}_i \in [\underline{k}, \overline{k}]} E_i[\widehat{u}_i], \quad s.t.\ \underline{k} \leq \widehat{k}_i \leq \overline{k}$$

and the optimal decision of the investor in the financial intermediary is $\widehat{k}_i = E_i[\widehat{A}] + \mu_1 - \mu_2$ with

- if $\underline{k} \leq E_i[\widehat{A}] \leq \overline{k}$ then $\mu_1 = \mu_2 = 0$ and $\widehat{k}_i = E_i[\widehat{A}]$,
- if $E_i[\widehat{A}] > \overline{k}$ then $\mu_1 = 0$ and $\mu_2 > 0$ and $\widehat{k}_i = \overline{k}$,
- if $E_i[\widehat{A}] < \underline{k}$ then $\mu_1 > 0$ and $\mu_2 = 0$ and $\widehat{k}_i = \underline{k}$,

so that

$$E_i[\widehat{u}_i] = \begin{cases} E_i[\widehat{A}]\underline{k} - \frac{1}{2}\underline{k}^2 & if \quad E_i[\widehat{A}] < \underline{k} \\ \frac{1}{2}E_i[\widehat{A}]^2 & if \quad \underline{k} \leq E_i[\widehat{A}] \leq \overline{k} \\ E_i[\widehat{A}]\overline{k} - \frac{1}{2}\overline{k}^2 & if \quad E_i[\widehat{A}] > \overline{k} \end{cases}$$

which compares with the utility of investors who prefer to invest directly in firms $E_i[\widetilde{u}_i] = \frac{1}{2}E_i[A]^2$, where we have already substituted the optimal decision $\widetilde{k}_i = E_i[A]$.

Since ξ is very small and $\widehat{k}_i$ is finite, then the value of $\overline{K}$ is arbitrarily close to the value of K which follows from Proposition 3. Since $k_i = \rho_3\overline{\theta} + \rho_4 x_i$, then $E_i[K] = \rho_3\overline{\theta} + \rho_4\frac{\gamma\overline{\theta}+\beta x_i}{\gamma+\beta}$.

There are three possible cases $\underline{k} > E_i[\widehat{A}]$, $\underline{k} \leq E_i[\widehat{A}] \leq \overline{k}$, and $E_i[\widehat{A}] > \overline{k}$. The case $\underline{k} \leq E_i[\widehat{A}] \leq \overline{k}$, may be (approximately) rewritten as $\underline{\Psi} \leq x_i \leq \overline{\Psi}$ with $\underline{\Psi} = \frac{\underline{k}+m-(1-\lambda)\frac{\widehat{\gamma}\overline{\theta}}{\widehat{\gamma}+\beta} - \lambda\rho_3\overline{\theta} - \lambda\rho_4\frac{\gamma\overline{\theta}}{\gamma+\beta}}{\frac{(1-\lambda)\beta}{\widehat{\gamma}+\beta} + \lambda\rho_4\frac{\beta}{\gamma+\beta}}$ and $\overline{\Psi} = \frac{\overline{k}+m-(1-\lambda)\frac{\widehat{\gamma}\overline{\theta}}{\widehat{\gamma}+\beta} - \lambda\rho_3\overline{\theta} - \lambda\rho_4\frac{\gamma\overline{\theta}}{\gamma+\beta}}{\frac{(1-\lambda)\beta}{\widehat{\gamma}+\beta} + \lambda\rho_4\frac{\beta}{\gamma+\beta}}$. In this case, $E_i[\widehat{u}_i] - E_i[\widetilde{u}_i] = \frac{1}{2}\left[E_i[\widehat{A}]^2 - E_i[A]^2\right]$ which is proportional to $E_i\left[(1-\lambda)\widehat{\theta} + \lambda\overline{K} - m\right]^2 - E_i\left[(1-\lambda)\theta + \lambda\overline{K}\right]^2$ and is (approximately) equal to

$$\begin{aligned}
&E_i\left[(1-\lambda)\widehat{\theta} + \lambda K - m\right]^2 - E_i\left[(1-\lambda)\theta + \lambda K\right]^2 \\
&= \left[(1-\lambda)\frac{\widehat{\gamma}\overline{\theta}+\beta x_i}{\widehat{\gamma}+\beta} + \lambda\left(\frac{\gamma}{\gamma+(1-\lambda)\beta}\overline{\theta} + \frac{(1-\lambda)\beta}{\gamma+(1-\lambda)\beta}\frac{\gamma\overline{\theta}+\beta x_i}{\gamma+\beta}\right) - m\right]^2 \\
&- \left[(1-\lambda)\frac{\gamma\overline{\theta}+\beta x_i}{\gamma+\beta} + \lambda\left(\frac{\gamma}{\gamma+(1-\lambda)\beta}\overline{\theta} + \frac{(1-\lambda)\beta}{\gamma+(1-\lambda)\beta}\frac{\gamma\overline{\theta}+\beta x_i}{\gamma+\beta}\right)\right]^2
\end{aligned}$$

which represents an inverted parabola.

There are two cases left, $\underline{k} > E_i[\widehat{A}]$ and $E_i[\widehat{A}] > \overline{k}$. For $\underline{\Psi} > x_i$, we obtain $E_i[\widehat{u}_i] - E_i[\widetilde{u}_i] = E_i[\widehat{A}]\underline{k} - \frac{1}{2}\underline{k}^2 - \frac{1}{2}E_i[A]^2$. This expression also represents an inverted parabola. For $x_i > \overline{\Psi}$, we also obtain an expression which represents an inverted parabola. The function $E_i[\widehat{u}_i] - E_i[\widetilde{u}_i]$ is continuous (in particular, it is continuous at $x_i = \underline{\Psi}$ and $x_i = \overline{\Psi}$).

Result. The function $E_i[\widehat{u}_i] - E_i[\widetilde{u}_i]$ has roots for m sufficiently low.

Proof. Consider the values of x_i for which $\underline{k} \leq E_i[\widehat{A}] \leq \overline{k}$. In this case, $E_i[\widehat{u}_i] - E_i[\widetilde{u}_i] > 0 \Leftrightarrow (1-\lambda)E_i\left[\widehat{\theta} - \theta\right] > m$ which holds for some x_i.■

There are two roots $\overline{\overline{x}}$ and $\overline{x}$. The cases $\underline{k} > E_i[\widehat{A}]$ and $E_i[\widehat{A}] > \overline{k}$ seldom happen for low $\underline{k}$ and large $\overline{k}$.
■

First, we restrict the set of those investors who can choose the financial intermediary to the less pessimistic investors. This is the most unfavorable setting for the impact of the financial intermediation, since the marginal effect would be stronger if other more pessimistic individuals could use the intermediary. This conservative approach is appropriate for our study, as we want to undoubtedly establish a role for financial intermediation.

In this simplified setting, the equilibrium investment decisions $\widetilde{k}_i$ and $\widehat{k}_i$ are linear. Order the individuals in the interval $[0,1]$ according to the size of their private signals, and denote the threshold investor who received signal $\overline{x}$ by $i(\overline{x})$. Define the set B_1 of investors who are slightly less optimistic than individual $i(\overline{x})$,

$$B_1 =]i(\overline{x}) - \xi, i(\overline{x})[$$

with small $\xi > 0$. We assume that only investors $i \in B_1$ have the option to choose the financial intermediary and, in equilibrium, these individuals invest through the intermediary.

Second, we restrict the set of those investors who can choose the financial intermediary to the more pessimistic investors, with

$$B_2 = \left]i(\overline{\overline{x}}), i(\overline{\overline{x}}) + \xi\right[$$

with $B \in \{B_1, B_2\}$.

We guess $\widetilde{k}_i = \rho_5\overline{\theta} + \rho_6 x_i + \rho_7 m + \rho_8$ and $\widehat{k}_i = \rho_9\overline{\theta} + \rho_{10} x_i + \rho_{11} m + \rho_{12}$, so that

$$\begin{aligned}
\overline{K} &= \int\limits_{[0,1]\setminus B} \widetilde{k}_i di + \int\limits_B \widehat{k}_i di \\
&= \int\limits_{[0,1]\setminus B} \left(\rho_5\overline{\theta} + \rho_6 x_i + \rho_7 m + \rho_8\right) di + \int\limits_B \left(\rho_9\overline{\theta} + \rho_{10} x_i + \rho_{11} m + \rho_{12}\right) di \\
&= \int\limits_{[0,1]\setminus B} \left(\rho_5\overline{\theta} + \rho_7 m + \rho_8\right) di + \int\limits_B \left(\rho_9\overline{\theta} + \rho_{11} m + \rho_{12}\right) di + \rho_6 \int\limits_{[0,1]\setminus B} x_i di + \rho_{10}\int\limits_B x_i di \\
&= (1-\xi)\left(\rho_5\overline{\theta} + \rho_7 m + \rho_8\right) + \xi\left(\rho_9\overline{\theta} + \rho_{11} m + \rho_{12}\right) + \rho_6(1-\xi)\theta + \rho_{10}\xi\overline{\overline{\overline{x}}} \\
&= (1-\xi)\left(\rho_5\overline{\theta} + \rho_6\theta + \rho_7 m + \rho_8\right) + \xi\left(\rho_9\overline{\theta} + \rho_{10}\overline{\overline{\overline{x}}} + \rho_{11} m + \rho_{12}\right)
\end{aligned}$$

with $\overline{\overline{\overline{x}}} = \frac{\int\limits_B x_i di}{\xi}$. Hence,

$$\begin{aligned}
\widetilde{k}_i &= E_i[(1-\lambda)\theta + \lambda\overline{K}] \\
&= (1-\lambda)\frac{\gamma\overline{\theta} + \beta x_i}{\gamma+\beta} + \lambda E_i\left[(1-\xi)\left(\rho_5\overline{\theta} + \rho_6\theta + \rho_7 m + \rho_8\right) + \xi\left(\rho_9\overline{\theta} + \rho_{10}\overline{\overline{\overline{x}}} + \rho_{11} m + \rho_{12}\right)\right] \\
&= (1-\lambda)\frac{\gamma\overline{\theta} + \beta x_i}{\gamma+\beta} + \lambda\left[(1-\xi)\left(\rho_5\overline{\theta} + \rho_6\frac{\gamma\overline{\theta} + \beta x_i}{\gamma+\beta} + \rho_7 m + \rho_8\right) + \xi\left(\rho_9\overline{\theta} + \rho_{10}\overline{\overline{\overline{x}}} + \rho_{11} m + \rho_{12}\right)\right] \\
&= \left((1-\lambda)\frac{\gamma}{\gamma+\beta} + \lambda(1-\xi)\rho_5 + \lambda(1-\xi)\rho_6\frac{\gamma}{\gamma+\beta} + \lambda\xi\rho_9\right)\overline{\theta} + \left((1-\lambda)\frac{\beta}{\gamma+\beta} + \lambda(1-\xi)\rho_6\frac{\beta}{\gamma+\beta}\right)x_i \\
&\quad + \left[\lambda(1-\xi)\rho_7 + \lambda\xi\rho_{11}\right] m + \lambda(1-\xi)\rho_8 + \lambda\xi\left(\rho_{10}\overline{\overline{\overline{x}}} + \rho_{12}\right)
\end{aligned}$$

and

$$
\begin{aligned}
\widehat{k}_i &= E_i\left[(1-\lambda)\widehat{\theta}+\lambda\overline{K}-m\right] = (1-\lambda)\frac{\widehat{\gamma}\overline{\theta}+\beta x_i}{\widehat{\gamma}+\beta}+\lambda E_i[\overline{K}]-m \\
&= (1-\lambda)\frac{\widehat{\gamma}\overline{\theta}+\beta x_i}{\widehat{\gamma}+\beta}+\lambda E_i[(1-\xi)\left(\rho_5\overline{\theta}+\rho_6\theta+\rho_7 m+\rho_8\right)+\xi\left(\rho_9\overline{\theta}+\rho_{10}\overline{\overline{\overline{x}}}+\rho_{11}m+\rho_{12}\right)]-m \\
&= (1-\lambda)\frac{\widehat{\gamma}\overline{\theta}+\beta x_i}{\widehat{\gamma}+\beta}+\lambda\left[(1-\xi)\left(\rho_5\overline{\theta}+\rho_6\frac{\gamma\overline{\theta}+\beta x_i}{\gamma+\beta}+\rho_7 m+\rho_8\right)+\xi\left(\rho_9\overline{\theta}+\rho_{10}\overline{\overline{\overline{x}}}+\rho_{11}m+\rho_{12}\right)\right]-m \\
&= \left((1-\lambda)\frac{\widehat{\gamma}}{\widehat{\gamma}+\beta}+\lambda(1-\xi)\rho_5+\lambda(1-\xi)\rho_6\frac{\gamma}{\gamma+\beta}+\lambda\xi\rho_9\right)\overline{\theta}+\left((1-\lambda)\frac{\beta}{\widehat{\gamma}+\beta}+\lambda(1-\xi)\rho_6\frac{\beta}{\gamma+\beta}\right)x_i \\
&\quad+\left[\lambda(1-\xi)\rho_7+\lambda\xi\rho_{11}-1\right]m+\lambda(1-\xi)\rho_8+\lambda\xi\left(\rho_{10}\overline{\overline{\overline{x}}}+\rho_{12}\right).
\end{aligned}
$$

The initial guesses are verified with

$$
\begin{aligned}
\rho_5 &= (1-\lambda)\frac{\gamma}{\gamma+\beta}+\lambda(1-\xi)\rho_5+\lambda(1-\xi)\rho_6\frac{\gamma}{\gamma+\beta}+\lambda\xi\rho_9 \\
\rho_6 &= (1-\lambda)\frac{\beta}{\gamma+\beta}+\lambda(1-\xi)\rho_6\frac{\beta}{\gamma+\beta} \\
\rho_7 &= \lambda(1-\xi)\rho_7+\lambda\xi\rho_{11} \\
\rho_8 &= \lambda(1-\xi)\rho_8+\lambda\xi\left(\rho_{10}\overline{\overline{\overline{x}}}+\rho_{12}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
\rho_9 &= (1-\lambda)\frac{\widehat{\gamma}}{\widehat{\gamma}+\beta}+\lambda(1-\xi)\rho_5+\lambda(1-\xi)\rho_6\frac{\gamma}{\gamma+\beta}+\lambda\xi\rho_9 \\
\rho_{10} &= (1-\lambda)\frac{\beta}{\widehat{\gamma}+\beta}+\lambda(1-\xi)\rho_6\frac{\beta}{\gamma+\beta} \\
\rho_{11} &= \lambda(1-\xi)\rho_7+\lambda\xi\rho_{11}-1 \\
\rho_{12} &= \lambda(1-\xi)\rho_8+\lambda\xi\left(\rho_{10}\overline{\overline{\overline{x}}}+\rho_{12}\right)
\end{aligned}
$$

so that

$$
\begin{aligned}
\rho_7 &= -\frac{\lambda\xi}{1-\lambda}, \rho_{11}=\rho_7-1 \\
\rho_6 &= \frac{(1-\lambda)\beta}{(\gamma+\beta)-\lambda(1-\xi)\beta} \\
\rho_{10} &= \frac{1}{(1-\lambda\xi)}\left[\frac{(1-\lambda)\beta}{\widehat{\gamma}+\beta}+\frac{\lambda(1-\xi)(1-\lambda)\beta^2}{\left[(\gamma+\beta)-\lambda(1-\xi)\beta\right]\gamma+\beta}\right] \\
\rho_8 &= \rho_{12}=\frac{\lambda\xi\overline{\overline{\overline{x}}}}{1-\lambda}\rho_{10} \\
\rho_5 &= \frac{1-\lambda+\lambda\left(1-\xi\right)\rho_6\left(1+\frac{\lambda\xi}{1-\lambda}\right)+\lambda^2\xi\left(1-\xi\right)}{1-\lambda\left(1-\xi\right)}\frac{\gamma}{\gamma+\beta}+\lambda\xi\frac{\widehat{\gamma}}{\widehat{\gamma}+\beta} \\
\rho_9 &= \lambda\left(1-\xi\right)\left(1+\frac{\rho_6}{1-\lambda}\right)\frac{\gamma}{\gamma+\beta}+\left[1-\lambda\left(1-\xi\right)\right]\frac{\widehat{\gamma}}{\widehat{\gamma}+\beta}.
\end{aligned}
$$

When $\xi \to 0$,

$$\rho_7 = 0, \rho_{11} = -1$$
$$\rho_6 = \rho_4$$
$$\rho_{10} = \left[\frac{(1-\lambda)\beta}{\widehat{\gamma}+\beta} + \frac{\lambda(1-\lambda)\beta^2}{[(\gamma+\beta)-\lambda\beta]\,\gamma+\beta} \right]$$
$$\rho_8 = \rho_{12} = 0$$
$$\rho_5 = \rho_3$$
$$\rho_9 = \lambda \left(1 + \frac{\rho_4}{1-\lambda}\right) \frac{\gamma}{\gamma+\beta} + (1-\lambda)\frac{\widehat{\gamma}}{\widehat{\gamma}+\beta}. \blacksquare$$

The proof is valid for any set $B = \left]i\,(x_0) - \frac{\xi}{2}, i\,(x_0) + \frac{\xi}{2}\right[$, with small $\xi > 0$ and $x_0 \in \left]\overline{\overline{x}}, \overline{x}\right[$.

A.5 Proof of Proposition 5

We proceed by steps. First, we compare the level of investment for those individuals who invest through the bank ($i \in B$) with what they would invest if there were no banks. Second, we compare the level of investment of those individuals who choose direct finance when there are banks ($i \in [0,1]\backslash B$), with their investment when there are no banks. Finally, we compare the level of aggregate investment and the individual investment decisions with and without coexistence.

Step 1. To show that $\int\limits_B \widehat{k}_i di > \int\limits_B k_i di$ we use the following Lemma.

Lemma 2 *The integral* $\int\limits_B \widehat{k}_i di$ *converges to*

$$(1-\lambda)\int\limits_B E_i[\widehat{\theta}]di + \lambda(1-\lambda)\int\limits_B\int\limits_B E_i\left[E_j[\widehat{\theta}]\right]djdi + \lambda^2(1-\lambda)\int\limits_B\int\limits_B\int\limits_B E_i\left[E_j\left[E_\iota[\widehat{\theta}]\right]\right]d\iota djdi + ...$$

$$-\int\limits_B mdi - \lambda\int\limits_B\int\limits_B mdjdi - \lambda^2\int\limits_B\int\limits_B\int\limits_B md\iota djdi - ... + \lambda\int\limits_B\int\limits_{[0,1]\backslash B} E_i\left[\widetilde{k}_j\right]djdi$$

$$+\lambda^2\int\limits_B\int\limits_B\int\limits_{[0,1]\backslash B} E_i\left[E_j\left[\widetilde{k}_\iota\right]\right]d\iota djdi + \lambda^3\int\limits_B\int\limits_B\int\limits_B\int\limits_{[0,1]\backslash B} E_i\left[E_j\left[E_\iota\left[\widetilde{k}_\tau\right]\right]\right]d\tau d\iota djdi + ...$$

Proof. We have $\int\limits_B \widehat{k}_i di = \int\limits_B E_i[(1-\lambda)\widehat{\theta} + \lambda\overline{K} - m]di = \int\limits_B \left((1-\lambda)E_i[\widehat{\theta}] + \lambda E_i[\overline{K}] - m\right)di$

$$= \int\limits_B \left((1-\lambda)E_i[\widehat{\theta}] + \lambda E_i\left[\int\limits_{[0,1]\backslash B}\widetilde{k}_j dj + \int\limits_B \widehat{k}_j dj\right] - m\right)di$$

$$= (1-\lambda)\int\limits_B E_i[\widehat{\theta}]di + \lambda\int\limits_B\int\limits_{[0,1]\backslash B} E_i\left[\widetilde{k}_j\right]djdi + \lambda\int\limits_B\int\limits_B E_i\left[\widehat{k}_j\right]djdi - \int\limits_B mdi$$

$$= (1-\lambda)\int_B E_i[\widehat{\theta}]di + \lambda\int_B\int_{[0,1]\setminus B} E_i\left[\widetilde{k}_j\right]djdi + \lambda\int_B\int_B E_i\left[(1-\lambda)E_j[\widehat{\theta}] + \lambda E_j[\overline{K}] - m\right]djdi - \int_B mdi$$

$$= (1-\lambda)\int_B E_i[\widehat{\theta}]di + \lambda\int_B\int_{[0,1]\setminus B} E_i\left[\widetilde{k}_j\right]djdi + \lambda(1-\lambda)\int_B\int_B E_i\left[E_j[\widehat{\theta}]\right]djdi + \lambda^2\int_B\int_B E_i\left[E_j[\overline{K}]\right]djdi - \lambda\int_B\int_B mdjdi - \int_B mdi\ .$$

Reorder the terms, replace $\overline{K}$ and iterate again,

$$= (1-\lambda)\int_B E_i[\widehat{\theta}]di + \lambda(1-\lambda)\int_B\int_B E_i\left[E_j[\widehat{\theta}]\right]djdi - \lambda\int_B\int_B mdjdi - \int_B mdi + \lambda\int_B\int_{[0,1]\setminus B} E_i\left[\widetilde{k}_j\right]djdi$$

$$+\lambda^2\int_B\int_B E_i\left[E_j\left[\int_{[0,1]\setminus B}\widetilde{k}_\iota d\iota + \int_B \widehat{k}_\iota d\iota\right]\right]djdi$$

$$= (1-\lambda)\int_B E_i[\widehat{\theta}]di + \lambda(1-\lambda)\int_B\int_B E_i\left[E_j[\widehat{\theta}]\right]djdi - \lambda\int_B\int_B mdjdi - \int_B mdi + \lambda\int_B\int_{[0,1]\setminus B} E_i\left[\widetilde{k}_j\right]djdi +$$

$$\lambda^2\int_B\int_B E_i\left[E_j\left[\int_{[0,1]\setminus B}\widetilde{k}_\iota d\iota\right]\right]djdi + \lambda^2\int_B\int_B E_i\left[E_j\left[\int_B \widehat{k}_\iota d\iota\right]\right]djdi\ .$$

Iterating n times and letting $n\to\infty$, the term in $\widehat{k}$ vanishes and we obtain the result. ■

Apply Lemma 2 to obtain

$$\int_B k_i di = (1-\lambda)\int_B E_i[\theta]di + \lambda(1-\lambda)\int_B\int_B E_i\left[E_j[\theta]\right]djdi + \lambda^2(1-\lambda)\int_B\int_B\int_B E_i\left[E_j\left[E_\iota[\theta]\right]\right]d\iota djdi + ...$$

$$+\lambda\int_B\int_{[0,1]\setminus B} E_i\left[k_j\right]djdi + \lambda^2\int_B\int_B\int_{[0,1]\setminus B} E_i\left[E_j\left[k_\iota\right]\right]d\iota djdi$$

$$+\lambda^3\int_B\int_B\int_B\int_{[0,1]\setminus B} E_i\left[E_j\left[E_\iota\left[k_\tau\right]\right]\right]d\tau d\iota djdi + ...$$

Compute

$$\int_B \widehat{k}_i di - \int_B k_i di =$$

$$(1-\lambda)\int_B E_i[\widehat{\theta}-\theta]di + \lambda(1-\lambda)\int_B\int_B E_i\left[E_j[\widehat{\theta}-\theta]\right]djdi + \lambda^2(1-\lambda)\int_B\int_B\int_B E_i\left[E_j\left[E_\iota[\widehat{\theta}-\theta]\right]\right]d\iota djdi + ...$$

$$-\int_B mdi - \lambda\int_B\int_B mdjdi - \lambda^2\int_B\int_B\int_B md\iota djdi - ... + \lambda\int_B\int_{[0,1]\setminus B} E_i\left[\widetilde{k}_j - k_j\right]djdi$$

$$+\lambda^2\int_B\int_B\int_{[0,1]\setminus B} E_i\left[E_j\left[\widetilde{k}_\iota - k_\iota\right]\right]d\iota djdi + \lambda^3\int_B\int_B\int_B\int_{[0,1]\setminus B} E_i\left[E_j\left[E_\iota\left[\widetilde{k}_\tau - k_\tau\right]\right]\right]d\tau d\iota djdi + ...$$

For sufficiently low ξ,

$$\int\limits_B \widehat{k}_i di - \int\limits_B k_i di \to$$

$$(1-\lambda)\int\limits_B E_i[\widehat{\theta}-\theta]di + \lambda(1-\lambda)\int\limits_B\int\limits_B E_i\left[E_j[\widehat{\theta}-\theta]\right]djdi + \lambda^2(1-\lambda)\int\limits_B\int\limits_B\int\limits_B E_i\left[E_j\left[E_\iota[\widehat{\theta}-\theta]\right]\right]d\iota djdi + ...$$

Since $E_i\left[\widehat{\theta}\right] > E_i[\theta]$ for $i \in B$, then $\int\limits_B \widehat{k}_i di - \int\limits_B k_i di > 0$.

Step 2. To show $\int\limits_{[0,1]\setminus B} \widetilde{k}_i di > \int\limits_{[0,1]\setminus B} k_i di$, apply Lemma 2 to both integrals to obtain

$$\int\limits_{[0,1]\setminus B} \widetilde{k}_i di - \int\limits_{[0,1]\setminus B} k_i di =$$

$$\int\limits_{[0,1]\setminus B} \left(\widetilde{k}_i - k_i\right) di = \lambda \int\limits_{[0,1]\setminus B}\int\limits_B E_i\left[\widehat{k}_j - k_j\right]djdi + \lambda^2 \int\limits_{[0,1]\setminus B}\int\limits_{[0,1]\setminus B}\int\limits_B E_i\left[E_j\left[\widehat{k}_\iota - k_\iota\right]\right]d\iota djdi + ...$$

The terms in $\widetilde{k}$ vanish as a result of Lemma 2. Since we proved in step1 that $\int\limits_B \widehat{k}_i di > \int\limits_B k_i di$, then

$$\int\limits_{[0,1]\setminus B} \widetilde{k}_i di - \int\limits_{[0,1]\setminus B} k_i di > 0 \Leftrightarrow \int\limits_{[0,1]\setminus B} \widetilde{k}_i di > \int\limits_{[0,1]\setminus B} k_i di.$$

Step 3. By steps 1 and 2, $\overline{K} > K$ and for a sufficiently low m, we obtain $\widehat{k}_i = (1-\lambda)E_i[\widehat{\theta}] + \lambda E_i[\overline{K}] - m > (1-\lambda)E_i[\theta] + \lambda E_i[K] = k_i$ for $i \in B$, and $\widetilde{k}_i = (1-\lambda)E_i[\theta] + \lambda E_i[\overline{K}] > (1-\lambda)E_i[\theta] + \lambda E_i[K] = k_i$.■

A.6 Proof of Proposition 6

For $i \in B$, individuals choose $\widehat{k}_i$ to maximize their expected utility. Hence,

$$\widehat{u}_i = E_i\left[\left[(1-\lambda)\widehat{\theta} + \lambda\overline{K} - m\right]\widehat{k}_i - \frac{1}{2}\widehat{k}_i^2\right] \geq E_i\left[\left[(1-\lambda)\widehat{\theta} + \lambda\overline{K} - m\right]k_i - \frac{1}{2}k_i^2\right]$$

where k_i is given in Proposition 3. It follows that

$$E_i\left[\left[(1-\lambda)\widehat{\theta} + \lambda\overline{K} - m\right]k_i - \frac{1}{2}k_i^2\right] > E_i\left[\left[(1-\lambda)\theta + \lambda K - m\right]k_i - \frac{1}{2}k_i^2\right]$$

since $E_i\left[\widehat{\theta}\right] > E_i[\theta]$ and $\overline{K} > K$. Moreover,

$$E_i\left[\left[(1-\lambda)\theta + \lambda K - m\right]k_i - \frac{1}{2}k_i^2\right] = E_i\left[u_i - mk_i\right],$$

so that $E_i\left[\widehat{u}_i\right] > E_i\left[u_i - mk_i\right]$ and $E_i\left[\widehat{u}_i\right] > E_i\left[u_i\right]$for sufficiently low m.

For $i \in [0,1] \backslash B$, individuals choose $\widetilde{k}_i$ to maximize their expected utility. Hence,

$$E_i\left[\widetilde{u}_i\right] = E_i\left[\left[(1-\lambda)\theta + \lambda\overline{K}\right]\widetilde{k}_i - \frac{1}{2}\widetilde{k}_i^2\right] \geq E_i\left[\left[(1-\lambda)\theta + \lambda\overline{K}\right]k_i - \frac{1}{2}k_i^2\right]$$

where k_i is given in Proposition 3. Since $\overline{K} > K$, $E_i\left[\widetilde{u}_i\right] > E_i\left[u_i\right]$.

Ex ante expected social welfare with coexistence is given by $E\left[\int\limits_{[0,1]\backslash B} E_i\left[\widetilde{u}_i\right]di + \int\limits_{B} E_i\left[\widehat{u}_i\right]di\right]$ which is larger than $E\left[\int\limits_{[0,1]\backslash B} E_i\left[u_i\right]di + \int\limits_{B} E_i\left[u_i\right]di\right]$, where u_i is the equilibrium utility obtained in Proposition 3. Since $E\left[\int\limits_{[0,1]\backslash B} E_i\left[u_i\right]di + \int\limits_{B} E_i\left[u_i\right]di\right] = E\left[\int_0^1 E_i\left[u_i\right]di\right] = E\left[\int_0^1 u_i di\right]$ by the law of iterated expectations, we obtain the result.■

Agglomeration and Industry Spillover Effects in the Aftermath of a Credit Crunch

Essay 3

January 2016

Abstract

This paper provides empirical evidence showing that industries with intense strategic complementarities exhibit stronger sensitivity to economic shocks. The 2009´s Portuguese credit crunch represents a negative shock for nonfinancial firms, which has created negative spillover effects among firms. Corporate investment declines significantly in industries with strong strategic complementarities following the onset of the crisis, controlling for firm fixed effects, time varying measures of financial constraints and investment opportunities. Consistent with a causal effect, the decline is greatest for firms in industries with strong strategic complementarities. To address sample selection concerns we consider several sample splits and apply a matching approach to find the best counterfactual, and confirm similar results.

Keywords: Banking, Firms Behaviour, Financial Crises, Coordination, Complementarities, Externalaties, Panel Data

JEL Classification Codes: G21, D22, G01, C72, D62, C23

1 Introduction

Do spillover effects among firms amplify economic shocks? Will firms reduce their output when their neighbors suffer negative shocks? Which firms suffer the most from spillover effects after an adverse shock? Do these spillover effects depend on the industry? Which industries suffer more from spillovers after a credit crunch? To answer these questions we consider a simple model in which we compare spillover effects among industries before and after a credit shock.

We are interested on a very particular type of spillover effects. First, we are interested in an environment in which operating firms are interdependent, with their success depending on the success of other operating firms—to put it more formally, we are interested on those spillover effects which are the source of strategic complementarities among firms. More specifically, we are interested in production externalities where the production of one firm increases the productivity of the others. Second, we are interested in production externalities which arise as a result of knowledge spillovers, labor market pooling, and input sharing (the three sources of external scale economies identified by Marshall, 1890). Third, we focus exclusively on intra-industry effects. The Venn diagram in Figure 1 clarifies the type of spillover effects analyzed in this paper.

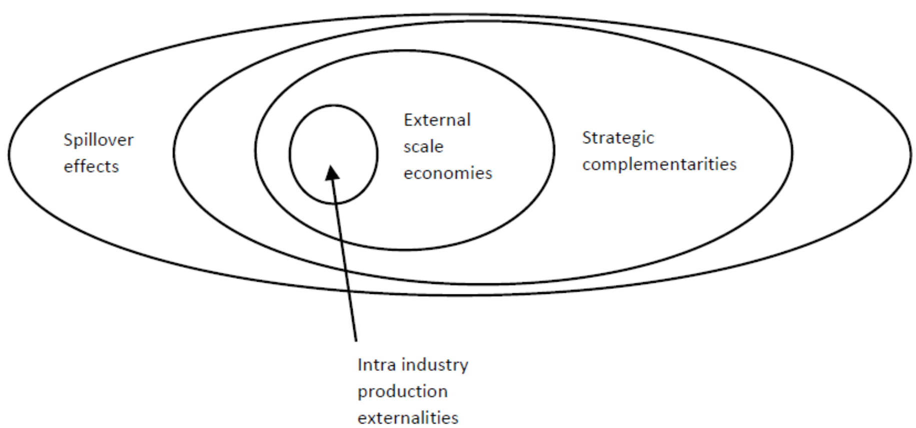

Figure 1: Venn diagram illustrating the different types of spillover effects. This paper focusses on intra-industry production externalities.

Our analysis is based on the premise (put forward in earlier work, such as Cooper and John, 1988; Angeletos and Pavan, 2004) that a significant fraction of firms benefits from the production of other firms in the economy. This interdependence can be generated by multiple channels. A firm's success depends on:

- The firms which use its outputs, and on those which supply its inputs (see, for example, Cooper and John, 1988).
- Industry-specific knowledge and information spillovers which take place in the industry (as in Carvalho and Voigtländer, 2014)
- Access to a large pool of skilled labor, which favors firm-worker matching.
- Its financial links, as bankruptcy from industry peers may have negative spillover effects.

As a result of these interdependencies, the firm's productivity and its profitability depend on its industry peers. It follows that the returns firms will make on borrowed capital will increase if other firms are able to obtain financing (either from markets or banks). When the aggregate production of the industry depends on bank financing, then a credit crunch will penalize the productivity of each individual firm *even if that individual firm does not see its credit being rationed*. A credit crunch will thus reduce the set of profitable investment opportunities of each firm.

We use the 2009's credit crunch in the Portuguese economy to identify the impact of a credit shock. Figure 2 depicts the evolution of bank lending in Portugal since January 2005 until the end of 2013. There was a sharp slowdown in the growth rate of bank credit from mid-2008 until mid-2010, with the annual growth becoming negative in 2009. The deleveraging of the Portuguese economy is related with the growing needs for bank capital and the liquidity problems faced by Portuguese banks after the global crisis in 2008.

In this paper we measure how banks' reluctance to extend loans to firms has compromised firms' investment. The hypotheses we take to the data are based on models with strategic complementarities among firms (Angeletos and Pavan, 2004 and 2007; Bebchuk and Goldstein, 2011; Jorge and Rocha, 2016). In theory, negative shocks might hinder firms which benefit from external scale economies, as reduced production by one firm hampers the productivity of the others. More specifically, theory suggests that shortages in the supply of external finance might hinder investment in those industries which display external scale economies. Moreover, such effects should be less severe in firms in industries without external scale economies.

To investigate these ideas, we employ a difference-in differences approach in which we compare firms' investment before and after the onset of the crisis as a function of how much they benefit from spillover effects (that is, if they belong to an industry which displays external scale economies), controlling for observable measures of external finance constraints and investment opportunities as well as firm fixed effects.

We are mostly interested in studying the role of strategic complementarities on worsening

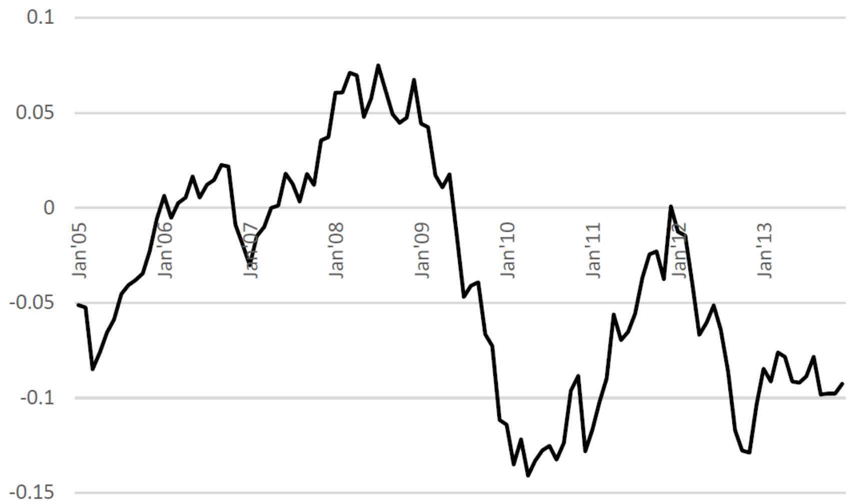

Figure 2: Evolution of bank lending in Portugal in the period 2005-13. This figure plots monthly growth rates of bank loans. Source: Banco de Portugal.

the impact of the credit crunch on investment. Yet, inferences may be confounded if variation in investment depends on unobserved *exogenous* variation in (*i*) credit conditions and, (*ii*) investment opportunities. There is a key distinction between *endogenous* and *exogenous* sources of variation in credit conditions and in investment opportunities, since endogenous changes in these variables are inevitable.

To fix ideas, consider the following example. A large number of investors are choosing how much to invest in a new sector. The profitability of the firms in this sector depends on an uncertain exogenous productivity parameter as well as on the aggregate investment. The investors thus have an incentive to align their choices. Moreover, decentralized individuals do not internalize the positive externality of their investment on the return of others, thus investing too little. In this setup, two channels will *endogenously* propagate a credit shock.

1. The coordination motive makes investment highly sensitive to the public assessment of the exogenous productivity parameter. Financial intermediaries mitigate the coordination problems, as monitoring the firm reduces the uncertainty about the firm's exogenous productivity parameter. Lower volatility entices less optimistic investors to invest, thus raising productivity and further stimulating investment. The other side of the coin is that an exogenous bank credit crunch reduces the ability of banks to monitor firms, thus aggravating the harmful effects of uncertainty and thus depressing investment and productivity (as in Jorge and Rocha, 2016).

2. The decision of an intermediary of whether to extend a loan to a given firm depends on the intermediary's assessment of the firm's exogenous productivity parameter and on its expectation of whether other intermediaries will lend money to other firms. This mechanism creates the potential for endogenous credit crunches with inevitable consequences on productivity (as in Bebchuk and Goldstein, 2011).

Our baseline specification, and the rest of our analysis described below, is designed to address the concerns about *exogenous* variation in credit conditions and in investment opportunities. To this purpose, we control for observable measures of external finance constraints (such as firm's debt) and investment opportunities (such as cash flow, sales, and whether the firm exports or not).

We are also concerned about heterogenous shocks in the supply of credit across industries, as banks might have cut credit more to some industries than to others—and have thus generated different effects across industries but which were not related with spillover effects. To this purpose, we use alternative measures of the bank credit shock (such as the total debt of the industry).

Moreover, we use several sample splits, in which we select a sample of firms which established relationships exclusively with banks which did not show reluctance to extend loans throughout the period 2006-2012, and a sample of exporting firms which were not affected by shocks in internal demand. Finally, we apply a matching approach to find the best counterfactual in the difference-in differences approach.

We find that operating firms which benefit from strategic complementarities were the most severely hit by the 2009's credit crunch. We measure the impact on annual investment as a ratio of assets for Portuguese manufacturing firms, and we compare the impact on industries with strategic complementarities to the impact on industries without complementarities. Consistent with the hypothesis that credit shocks are amplified in industries with strategic complementarities, we find that firms in industries with strategic complementarities reduce their investment by more than firms in other industries. Our final estimate suggests that firms with strategic complementarities reduce (on average) their annual investment (as a fraction of assets) by 3.3 percentage points more than firms without strategic complementarities following the onset of the 2009's credit crunch.

Evidence of spillover effects after credit shocks (for industries with complementarities) has important implications not only for borrowers, but also for policy makers (as emphasized by Bebchuk and Goldstein, 2011; Jorge and Rocha, 2016).

Review of the literature. Our article is related to several disjoint bodies of literature. The importance of financial constraints for investment decisions is a classic in finance, with extensions to macroeconomic theory (as, for example, Hoshi, Kashyap, and Scharfstein, 1991; Kaplan and Zingales, 1997; Bernanke, Gertler and Gilchrist, 1999; Dell'Ariccia, Detragiache and Rajan, 2008). More specifically, Kashyap and Stein, (1994) and (2000) highlight the role of the bank lending channel. Our paper contributes to this literature by emphasizing the role of strategic complementarities among firms.

The 2007-2009 global financial crisis has been used as an experimental field to study the effects of banks' distress on credit supply (as, for example, Tong and Wei, 2008; Ivashina and Scharfstein, 2010; Campello, Graham and Harvey, 2010). Within this literature, the paper relates to Lemmon and Roberts (2010), Duchin, Ozbas and Sensoy (2010) and Almeida, Campello, Laranjeira and Weisbenner (2011), who document a reduction in corporate investment as a consequence of supply shocks to external financing. Our results provide evidence that 2009 credit shock in Portugal had both direct and indirectly real effects on firms' investment.

A number of papers in the financial literature have used bankruptcy as an instrument to identify channels for spillover effects among firms. Lang and Stulz (1992) and Ferris, Jayaraman and Makhija (1997) document spillover effects of bankruptcy filings on investors of industry peers. Hertzel, Li, Officer and Rodgers (2008) examine bankruptcy contagion effects along the supply chain of filing firms, while Boone and Ivanov (2012) define proximate non-filing firms as strategic alliance partners. Jorion and Zhang (2007) and Hertzel and Officer (2012) document bankruptcy contagion effects on industry capital providers. Addoum, Kumar, Le and Niessen-Ruenzi (2015) document that, following the bankruptcy of a geographically proximate firm, firms that are located geographically near the bankrupt firm reduce their investment expenditures. They investigate channels for contagion related with executives' career concerns, and document that local firms experience worse credit conditions if a local firm files for bankruptcy. Although this literature presents *one* channel for spillover effects among individual firms, there are many other sources of strategic complementarities.

Instead, our priors are rooted on a strong theoretical background which identifies mechanisms which go beyond specific events like bankruptcy and have important macroeconomic implications. We base our empirical analysis on the following theoretical contributions.

In a model with strategic complementarities and bank lending, Bebchuk and Goldstein (2011) show that firms are vulnerable to credit market freezes. Banks avoid lending to firms out of self-fulfilling fear that other banks would withhold loans to firms, thus causing their default. Our paper links firms' investment decisions and strategic complementarities and our results highlight the importance of intra-industry production externalities on firms' investment decisions.

In a model with production externalities where the production of one firm increases the productivity of the others, Jorge and Rocha (2016) suggest that bank lending is more important when strategic complementarities are most prevalent, so that a credit contraction should have different impact across industries and geographical areas. Current paper suggests that spillover effects are important to understand how credit supply shock spread through industries

The remainder of the paper is organized as follows. We provide details on the role of strategic complementarities in firms' investment in Section 2. Section 3 examines the shock in bank credit in Portugal in 2009. Section 4 presents the empirical strategy, stating our hypotheses, the baseline specification and robustness tests implemented. Section 5 presents data and research methods. In Sections 6 and 7 we present and discuss our results in detail. Some conclusions are offered in the final section.

2 The role of strategic complementarities in firm's investment

Industries with production externalities are one example in which strategic complementarities play a prominent role. In these industries, spillover effects among firms will raise the productivity of an entire industry. One justification for the existence of production externalities is Alfred Marshall (1890)'s concept of external economies.

The benefits of external scale economies depend on the level of output of the industry, thus implying that a reduction in the output of the industry will have a negative impact on firms' productivity. Since productivity is a key determinant of investment opportunities, it follows that a negative shock in output is likely to reduce investment since capital will be less productive. Operating companies will face more difficulties in an environment in which other operating firms reduce their output.

The 2009's credit market freeze in the Portuguese economy represents an *aggregate* economic shock. The theory suggests that shortages in the supply of external finance might hinder output in those industries which display external scale economies. For example, Bebchuk and Goldstein (2011) study an economy with positive spillovers among operating firms in which their success depends on the ability of other operating firms to obtain credit. In such an economy, a credit crunch may arise in which banks abstain from lending to firms with good projects because of their self-fulfilling fear that other banks will not be making such loans. Such credit crunch leads to a fall in investment, since companies will not be able to succeed in an environment in which other firms fail to obtain finance.

Theory suggests that the effects of economic shocks (namely, a credit crunch) on output and investment will be stronger in those industries which display intense external scale economies. A shock will have minor impact on those operating firms which do not benefit from strategic complementarities. For example, Figure 2(b) in Jorge and Rocha (2016) suggests that firms in industries with intense complementarities are more sensitive to credit rationing.

We compare the impact of bank lending on investment across firms which belong to industries with and without strategic complementarities. Two operating firms with identical economic and financial conditions could react differently to the same credit contraction in terms of their investment decisions, depending on their levels of strategic complementarities. We expect the firm with intense complementarities to have the largest reaction, controlling for its intrinsic conditions.

Figure 3 shows the evolution of capital between 2006 and 2012 among two representative groups of Portuguese manufacturing firms: one group includes industries which display intense complementarities, and the other group includes firms which display minor strategic complementarities. Using 2008 as the reference year, the figure shows that the evolution of capital is similar among both groups until 2008. After 2009, though, capital falls sharply for those operating firms which benefit from external scale economies, whereas it remains relatively stable for the other group of firms. The evolution of capital suggests that operating firms which benefit from strategic complementarities were the most severely hit by the 2009's credit crunch.

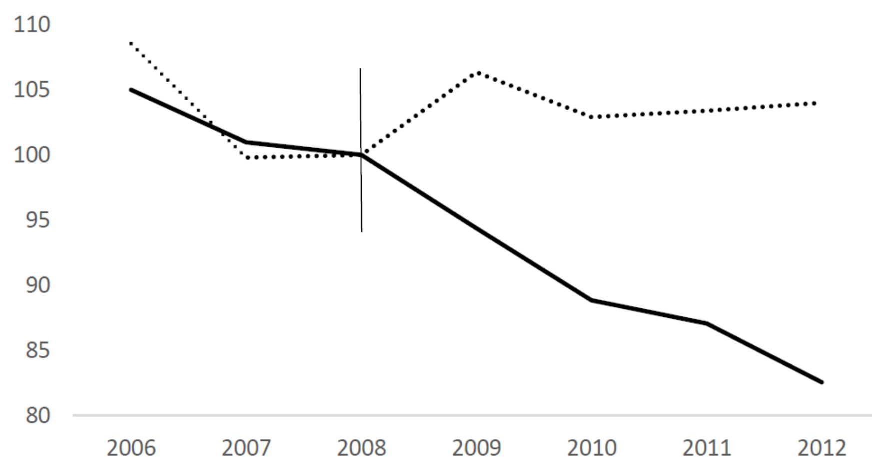

Figure 3: Effect of the 2009 credit shock on firms' ratio of capital over assets. The figure plots the evolution of capital over assets among firms which belong to industries with strategic complementarities (solid line) and firms which belong to industries without strategic complementarities (dotted line). The reference year is 2008, in which the ratio of capital over assets takes the value 100. Capital is the sum of tangible fixed assets plus depreciations.

2.1 Identifying firms with strategic complementarities

External scale economies, internal to the industry but external to the firm, are a source of increasing returns for individual firms and create strategic complementarities.

Agglomeration is widely recognized as a source and result of external scale economies. We use location theory (as, for example, in Ellison and Glaeser, 1997) to identify those firms which benefit from external scale economies. The theory distinguishes three different sets of factors driving the firm's location decision problem: external economies, costs of the factors of production like labour and capital, transportation costs, and natural advantages. External economies arise from (i) knowledge spillovers, labor market pooling, and input sharing, and (ii) urbanization economies.

The literature has developed a number of location coefficients which quantify those external scale economies that result from the spatial concentration of firms of a particular industry in a given region and that are internalized by firms of that particular industry (see, for example, Ellison and Glaeser, 1997; Guimarães, Figueiredo, and Woodward, 2007). Their basic principle is to measure the discrepancy between the distribution of regional employment in a particular industry against the regional distribution of the overall employment. Examples of industries with high geographic concentration are high-tech industries in Silicon Valley, the auto industry in Detroit, the entertainment industry in Hollywood, or investment banking in London. We use the DM index proposed by Guimarães et al. (2007), which controls for:

- Randomness in location decisions, which naturally generates some clustering.
- Industry concentration, which also creates geographical concentration. The high geographical concentration in industries such as petroleum refining or cement and related products is almost entirely explained by industrial concentration (and thus by internal returns to scale) rather than by external scale economies associated with firms' clustering.
- Market factors, such as wages, land costs, market accessibility or transportation costs, which may generate geographical concentration but are not directly related with external economies.
- Urbanization economies. Controlling for this factor is a rather conservative approach, which is likely to reduce the significance of our results. Knowledge-intensive industries thrive on the clustering of workers who share ideas and expertise and is the source of external scale economies.

Since external scale economies are one source of strategic complementarities, we classify operating firms as "firms benefiting from strategic complementarities" if they belong to those industries which display external scale economies. More specifically, we use the DM index as a proxy for strategic complementarities, since firms belonging to industries with high localization indices are likely to benefit from external scale economies.

We distinguish firms which belong to industries with strategic complementarities and in which spillover effects are important, from those firms which belong to industries in which spillover effects are minor. To operationalize this distinction, we divide firms into two groups according to the DM index of their industry. The first group includes firms from industries with high DM indices (the proxy for strong strategic complementarities), whereas the second group includes firms in industries with low DM indices (and which do not benefit from strategic complementarities)—see the appendix for details.

3 The shock in bank credit

Figure 2 depicts the evolution of bank lending in Portugal since January 2005 until December 2013. There was a sharp slowdown in the growth rate of bank credit from mid-2008 until mid-2010, with the annual growth becoming negative in 2009. The deleveraging of the Portuguese economy is related with the growing needs for capital and liquidity problems faced by Portuguese banks after the global crisis in 2008. Evidence of tighter lending in 2009 abounds, and we compare firms' investment before and after this year.[1] Banks are the main source of financing for Portuguese small and medium size firms.

Banks displayed considerable reluctance to extend loans to firms and compromised their ability to invest. Figure 4 shows the effect of the 2009 credit shock on firms' investment decisions. We run separate panel regressions for firms belonging to the group with strategic complementarities and for the firms in the group without strategic complementarities. We regress investment on a set of year dummies, controlling for firm fixed effects. Firms in the group with strategic complementarities suffer a steep reduction on their investment after 2008, confirming the strong impact of the 2009's shock, whereas firms without strategic complementarities do not experience a significant reduction in investment up to 2011. Our empirical strategy consists of measuring the differential reduction in investment for both groups.

4 The empirical strategy

We study the role of strategic complementarities (a specific case of spillover effects) in amplifying the impact of economic shocks. For this purpose, we evaluate if a shock has different impact on the two groups of firms considered. Formally, we test the hypothesis that the group with strong complementarities is more sensitive to the shock. More specifically, we use the 2009's credit crunch to compare the impact of spillover effects on firms' investment decisions.

To analyze the impact of the spillover effects after the shock, we employ a difference-in differences approach in which we compare firms' investment before and after the onset of the credit crunch as a function of their degree of strategic complementarity, and controlling for observable measures of external finance constraints and exogenous investment opportunities as well as firm fixed effects.

[1] For an overview of the 2009's credit crunch see Antunes and Martinho (2012). We chose the year of 2009 to define the point in time for the beginning of the credit contraction, instead of 2011, avoiding the peak of the economic recession in Portugal and the confounding factors which derive from the Memorandum of Understanding.

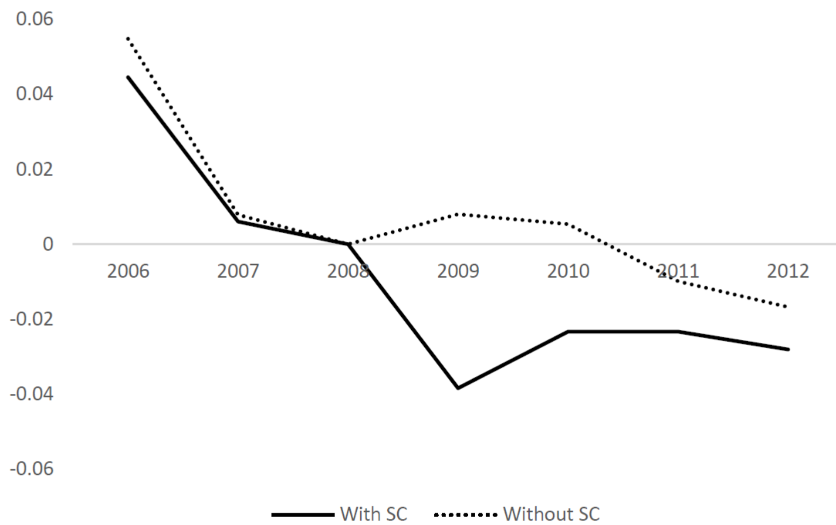

Figure 4: Effect of the 2009 credit shock on firms' ratio of investment over assets. We run separate panel regressions for firms with and without strategic complementarities of the investment over assets on the set of year dummies, controlling for firm fixed effects. The figure plots the coefficients obtained for the year dummies (2008 is the omitted year).

Our baseline specification regresses firm-level annual investment over 2006-2012 on a dummy variable for whether the year in question is after the shock, on a dummy variable for whether the firms belongs to the group with strategic complementarities, and on the interaction of the two dummy variables. The coefficient on the interaction term measures the differential impact of the credit shock on the two groups of firms.

The control variables used are total debt to account for external finance constraints, and cash flow, sales and exporter activity to account for exogenous investment opportunities.[2] Firm fixed effects subsume the dummy for the groups of firms (because the groups are fixed over time) and control for time-invariant heterogeneity across firms. Standard errors are clustered at the firm level to correct for within-firm residual correlation.

We conduct several additional robustness tests to address concerns that our results may be due to confounding effects. These include an alternative to identify the credit contraction shock, dealing with sample selection problems, and applying a matching approach to find the best counterfactual in the difference-in differences approach.

[2] Total debt is correlated with loan supply, and it is likely to be correlated with loan demand too. If this is the case, adding total debt to the regression will take explanatory power away from the interaction term. We will thus obtain a conservative estimate for the coefficient which measures the differential impact of the credit shock on the two groups of firms. Since it's difficult to obtain an instrument for loan supply, we make a sample slipt with banks which did not restrict credit (see below).

5 Data and research methods

We collect data from Sabi for Portuguese firms and for the period between 2006 and 2012, thus covering both crisis and pre-crisis years.[3] Sabi includes information about end-of-year balance sheets, income statements, and banking relationships, and comprehends (almost) all Portuguese firms. Our unit of observation is the firm-year pair.

We collect data on active firms with available accounting information, and restrict the selection to manufacturing industries. We sort firms into two groups, as we distinguish a group of firms in industries with large DM indices from a group of firms with low indices. We only use those industries with extreme values of the DM index, so as to make a clear distinction between the group with strategic complementarities and the group without strategic complementarities. Following Guimarães et al. (2007), we consider the top 19 industries with the highest DM indices, as well as the 16 industries in the lower end of the ranking. Industries with and without strategic complementarities are those with DM index measured by Guimarães, Figueiredo and Woodward 2007 (excluding for industries in petroleum refining, shipbuilding and repairing, sea products processing, tobacco, and recycling of non-metallic products). Industries with strategic complementarities are those with DM index above 0.026, and industries without strategic complementarities are those for which DM index is zero or not significantly different from zero at 95% confidence.

Departing from the list of industries presented by Guimarães et al. (2007), we exclude five industries because it is likely that their location depends on natural resources and not on spillover effects caused by strategic complementarities. These industries are petroleum refining, shipbuilding and repairing, sea products processing, tobacco, and recycling of non-metalic products. For example, agglomeration in shipbuilding and repairing naturally arises near seaports.

Finally, we restrict our sample to small and medium size firms (less than 250 employees) and we exclude micro firms with less than 10 employees (to guarantee reliable data). Using these filters, we collect data for 984 firms in the group with large DM indices and 240 firms in the group with low DM indices. The panel is not balanced, as only 730 firms have data for the 7 years. See the appendix for details.

Following much of the investment literature (as, for example, Duchin et al., 2010; Ivashina and Scharfstein, 2010; Almeida et al., 2011) we measure investment as capital expenditures divided by total assets. Capital expenditures in year t are calculated as the difference between

[3] Sabi database is the subset of the Amadeus database for European firms, and similar to Compustat for American firms.

"fixed tangible assets plus depreciations" in year t and the amount of "fixed tangible assets" in year $t-1$.

Table 1 reports summary statistics for firm-year units from 2006 until 2012. Panel A of Table 1 includes information on all observations in our sample, of which 6154 observations are on firms with strategic complementarities and 1490 observations are on firms without strategic complementarities. Panel A provides mean, standard deviation, minimum and maximum for several variables. Panel B of Table 1 distinguishes both groups of firms, and provides means, and difference-in-means tests for both groups.

The average values for variables like the ratio of capital, total debt, and cash-flow over assets show that the differences between the two groups are economically small, and the difference in investment between the two groups is not economically or statistically significant. There is a substantial difference in sales and assets, thus implying that the group of firms without strategic complementarities includes larger firms and suggesting that firms in this group benefit from internal scale economies.

6 Results

6.1 Preliminary results

Table 2 presents results for the two groups of firms (with and without strategic complementarities) in which we compare investment before the onset of the crisis to investment after. In the comparison, we average each firm's time series into two sample means—one for the period 2006-2008, which we label as "before the crisis", and one for the period 2009-2012, which we label as "after the crisis". Subsequently, we average the firms' sample means for each combination group-period. The table reports whether the differences in average investment between groups for each period are statistically significant.

The table shows that investment decreases by one-half for the group of firms with strategic complementarities. Although the reduction in investment for firms without strategic complementarities is statistically significant, it is substantially smaller from the economic point of view. Overall, results are consistent with our main hypothesis that tight credit conditions hurt more firms with strategic complementarities. In the analysis which follows, we investigate these patterns with more detail.

6.2 Baseline regressions

To quantify the impact of strategic complementarities on investment for both groups of firms *after the credit shock*, the analysis relies on the following difference-in differences specification.

$$INV_{it} = \alpha_1 + \beta_1 CC_t + \beta_2 SC_i + \beta_3 CC_t.SC_i + \boldsymbol{\beta}_4 W_{it} + \eta_t + \eta_i + \epsilon_{it} \tag{1}$$

where INV_{it} measures the investment of firm i in period t, CC_t stands for "credit contraction" and takes a value of 1 in the period "after" the shock (the period from 2009 through 2012) and 0 in the period "before" the shock (the period from 2006 through 2008), SC_i is a dummy variable which takes unit value for those firms which belong to the group with large DM indices and zero otherwise, the interaction term $CC_t.SC_i$ takes the value of 1 in the period of the credit contraction if the firm has strategic complementarities and zero otherwise, and W_{it} is a vector of control variables (cash flow, net sales variation, total debt, and a dummy variable which takes a value of 1 if firm i is exporter), η_t is a set of time dummies and η_i represents firm fixed effects.

The validity of the difference-in differences approach relies on satisfying the parallel trend assumption. When applied to equation (1), this assumption requires that the dependent variable would have followed the same trend for both groups (with and without strategic complementarities) in the absence of the credit shock.

Figure 5 plots the time series for investment for both groups of firms, with both series indexed to 100 in 2008. The figure shows a clear message: the trends in both groups are nearly identical until 2008, whereas in 2009 there is a clear break. After the onset of the crisis, the time series for investment by firms with large DM indices continues its downward trajectory, whereas investment for the group of firms with low DM indices grows in 2009 and later returns to the 2008 level. Such evolution in investment for both groups suggests that the parallel trend assumption applies.

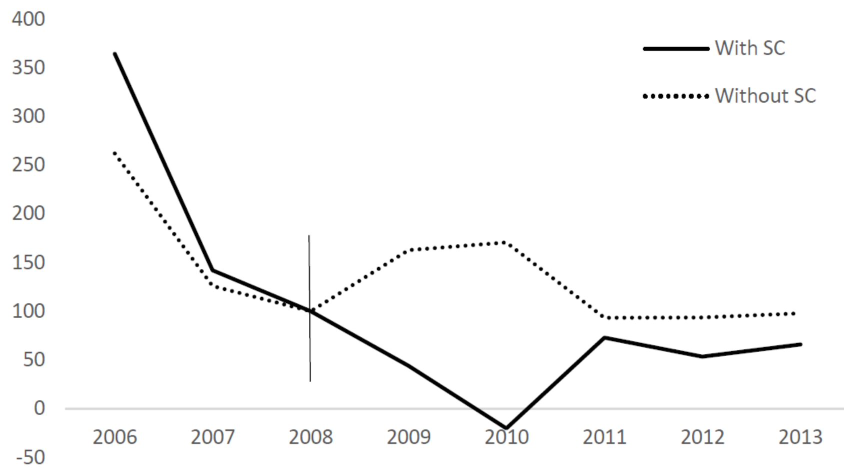

Figure 5: The parallel trend assumption. The figure plots the evolution of the ratio investment over assets for firms with strategic complementarities (solid line) and firms without strategic complementarities (dotted line). The reference year is 2008, in which the ratio of investment over assets takes the value 100.

6.2.1 Baseline results

Table 3 presents the estimates of regression equation (1). The key coefficient of interest is β_3—the coefficient on the interaction term—which measures the impact of the credit shock on the investment of firms with strong strategic complementarities. Standard errors are clustered at the firm-level to correct for within-firm residual correlation.

Columns (1) and (2) do not include controls or the dummy variable for strategic complementarities, but include fixed effects and a dummy variable for the credit contraction. Column (1) presents the basic patterns of investment. We find that, on average, annual investment as a fraction of assets declined by 3.71 percentage points following the onset of the credit shock, which compares with the unconditional mean of 5 percent.[4]

Column (2) includes the interaction term. Following the onset of the credit shock, annual investment (as a fraction of assets) declined by 2.09 percentage points more for firms with strategic complementarities. The coefficient on the interaction term shows that the decline in investment is economically large and statistically significant for firms with strategic complementarities, thus establishing a role for spillover effects in the aftermath of the credit shock.

The remaining columns include the control variables. Column (3) considers random effects and column (4) includes firm fixed effects. The coefficient on the dummy for strategic

[4]Banco de Portugal reports a fall of 16% in the gross fixed capital formation for the same period.

complementarities in column (3) suggests that firms with strategic complementarities invest around 1 percentage point more than firms without strategic complementarities. Yet, the Hausman test unambiguously rejects the existence of random effects. Column (4) considers the existence of fixed effects, and shows that the differential effect between firms with and without strategic complementarities increases to about 2.12 percentage points, and the effect remains highly statistically significant.

We use the firm's total debt (as a fraction of assets) in year t to control for external finance constraints. Yet, variation in firm's total debt as the crisis unfolds may be related to unobserved changes in its investment opportunities—to some extent, total debt is endogenous to the choices made by the firm. We remove these changes from our specification by using (only) the total debt over assets *before* the crisis. This is equivalent to using instrumental variables, assuming that the ratio of debt to assets before the crisis is not correlated with unobserved within-firm changes in investment opportunities after 2008. In Column (5) from Table 3, we repeat the exercise in column (3) replacing debt over assets in each year by the debt over assets in 2006. The table documents economically equivalent results, since the coefficient on the interaction term does not change substantially between the two alternative difference-in-differences specifications.

The estimates in column (6) include time dummies to control for aggregate shocks (which subsume the CC_t variable) together with firm fixed effects. The estimate of the differential impact increases to 2.17 percentage points and continues to be highly statistically significant.

In the previous columns we have compared the effects in the period 2006-2008 with the effects in period 2009-2012, which we have labeled as the periods "before" and "after" the shock. Instead, in column (7) we compare the effects in 2008 with the effects in 2009. To this purpose, we consider time dummies which take a value of 1 in year t and 0 otherwise, with the base year being 2008 (i.e. there is no dummy variable for year 2008). We interact the strategic complementarities dummy SC_i with the time dummies, and the key interaction term multiplies the dummy variable SC_i with the time dummy for year 2009. Results show that from the end of 2008 until the end of 2009, firms with strategic complementarities reduce (on average) their investment (as a fraction of assets) by 4.45 percentage points more than firms without strategic complementarities. The effect is statistically significant, thus reinforcing the economic significance of spillover effects.

6.3 Robustness checks: an alternative to identify the shock

We now address potential concerns with our baseline specification. First, we have defined the year 2009 as the year of the shock but there may be some concerns that this may not be the correct year—that is, the dummy CC_t does not correctly identify the economic shock. We have tested with other years, and the year 2009 is the year with the largest economic effects and with the most statistically significant coefficients. For example, results on column (7) in Table 3 confirm that 2009 is the year of the credit shock, since it has the largest estimated coefficient (in absolute terms) on the interaction term as well as the most significant. Still, it could be that the credit shock does not hit all industries simultaneously, and the spillover effects from strategic complementarities hit firms at different times.

Another concern is that the initial credit shock hits industries heterogeneously, since banks might have cut credit more to some industries than to others (and have thus generated different spillover effects across industries). In this case, a dummy variable will not fully capture the richness of the information found in the data.

To address these concerns, we change the definition of the variable which proxies the credit contraction. We repeat the baseline specification, replacing the dummy variable for the periods "before" and "after" the shock with a variable which measures the *evolution of credit for each industry.* We hope to identify the spillover effects which derive from the credit contraction for each particular industry.

We use the total debt (normalized by assets) of an industry as a proxy for the industry's bank credit. We sum the total debt of the firms belonging to a given industry to obtain the total debt of the industry— the debt reported in firms' balances sheets is closely related with bank credit since most of the credit to SMEs is granted by banks. Being a continuous variable which takes values for all years in our sample, the new variable solves our two concerns.

We apply a difference-in differences specification similar to equation (1), where we replace the variable CC_t by the variable $\Delta Debt_{it}$ which measures the difference between the values of year t and of year $t-1$ for the total debt of the industry (normalized by assets) to which firm i belongs (the credit to the industry to which firm i belongs falls as long as the variable $\Delta Debt_{it}$ takes negative values).

$$INV_{it} = \alpha_1 + \beta_1 \Delta Debt_{it} + \beta_2 SC_i + \beta_3 \Delta Debt_{it}.SC_i + \boldsymbol{\beta}_4 W_{it} + \eta_t + \eta_i + \epsilon_{it}$$

All other variables are as defined earlier. The interaction term $\Delta Debt_{it}.SC_i$ combines the variation in industry's debt with its strategic complementarity level, and we want to evaluate if the coefficient β_3 is positive and statistically significant. Column (1) in Table 4 reports the results for the estimated equation.

The results are broadly consistent with the previous results. The coefficient on the interaction term β_3 suggests that a reduction of 1 percentage point in the ratio of debt variation over assets for an industry with strategic complementarities, on average leads to a decline in firms' annual investment as a fraction of assets equal to 0.172 percentage points more than for industries without complementarities.

6.4 Robustness checks: sample splits

The regression model (1) was specified according to our theoretical priors, and we have added controls to the specification to capture additional sources of firm heterogeneity. But the inclusion of controls in the regression per se does not address the fact that the two groups being compared may have very different characteristics (see, for example, Heckman et al., 1998). When the control variables have poor distributional overlap, one can improve the estimation of group differences by estimating the model for more homogenous groups of firms.

Motivated by the potential sensitivity of our results to our sample, we estimate the model for appropriately selected subsamples. For the same reason, we will also conduct our analysis combining a difference-in differences approach with the use of matching estimators.

6.4.1 Handling a possible sample selection problem

One obvious concern about our identification strategy is the sample selection problem, which could arise from the possible migration by firms from those banks which have restricted their loans to those banks which have not. Since 2009, "good" firms could have migrated from banks which have restricted their credit or, instead, these banks could have "cherry-picked" the "good" firms. In any of these cases, the portfolio of banks which restricted their credit after 2009 represents a biased sample. For the same reason, the set of firms which has migrated among banks is also a biased sample.

For these reasons, we focus on those firms which have worked exclusively with banks which were more willing to extend loans to firms in the period 2009-2012. This strategy alleviates concerns about sample selection, such as bank-firm sorting.

The next step is to identify those banks which were less reluctant to extend loans to firms after 2009. Capital adequacy ratios have a major impact on the willingness of banks to grant credit (see, for example, Bebchuk and Goldstein 2011).

In 2012, given the increasing capital requirements imposed to Portuguese banks, some of the biggest Portuguese banks were required to ask for capital state support—this possibility had already been planned in the Memorandum of Understanding. Augusto and Felix (2014) analyzed the effects of this recapitalization in the period between 2010 and 2013 and its effect on firms' credit access, concluding that these bailout operations contributed to an increase in credit supply, that is, they prevented an even sharper break of loan growth rates.

Portuguese banks Banco Comercial Português, Banco Português de Investimento, and Caixa Geral de Depósitos (the state-owned bank) were bailed out in June 2012, and Banco Internacional do Funchal in December 2012 by the issuance of contingent convertible bonds, which allowed these banks to comply with minimum capital requirements defined by European Banking Authority and by Banco de Portugal. This recapitalization operation was necessary to reinforce banks' capital base, in a scenario of adverse macroeconomic conditions and compression of their net interest margins.

The depletion of capital is a long process and it is likely that banks may have started to face problems as early as 2009. This would imply that bank lending has been affected since 2009. Figure 6 depicts the evolution of total bank loans granted by each of the four largest Portuguese banks, and shows that the only bank that was not bailed out (Banco Espírito Santo) was precisely the one that has restricted its lending by less.[5]

Sabi contains information about the bank relationships of each individual firm for each year. We divide our sample of firms into three distinct groups: (*i*) firms which worked only with non-bailed out banks, (*ii*) firms which worked only with bailed out banks, and (*iii*) firms which worked with both types of banks. The reference period to build these three groups was 2006-2012. Figure 7 plots the evolution of the average values of the total debt difference between two consecutive years (as a fraction of assets) for the three distinct groups considered. The debt of firms which only worked with bailed out banks suffered a severe decrease in 2009, whereas the total debt of firms which worked exclusively with non-bailed out banks remained almost constant over time.

Having in mind Figures 6 and 7 , we consider a sample of firms which have obtained bank loans during the period 2006-2012 exclusively from banks which were not bailed out. Column (2) in Table 4 reports the estimates of equation (1) for the restricted sample. The results in the sub-sample reinforce the results in the baseline regressions. On average, firms with strategic complementarities reduce their investment (as a fraction of assets) by 6.08 percentage points more than firms without complementarities following the 2009 credit crunch.

[5] Caixa Geral de Depósitos, the state owned bank, exihbits a lending behaviour similar to Banco Espírito Santo, but anecdotal evidence points out that Caixa Geral de Depósitos has made a large effort to offset the decrease in aggregate bank lending after 2008.

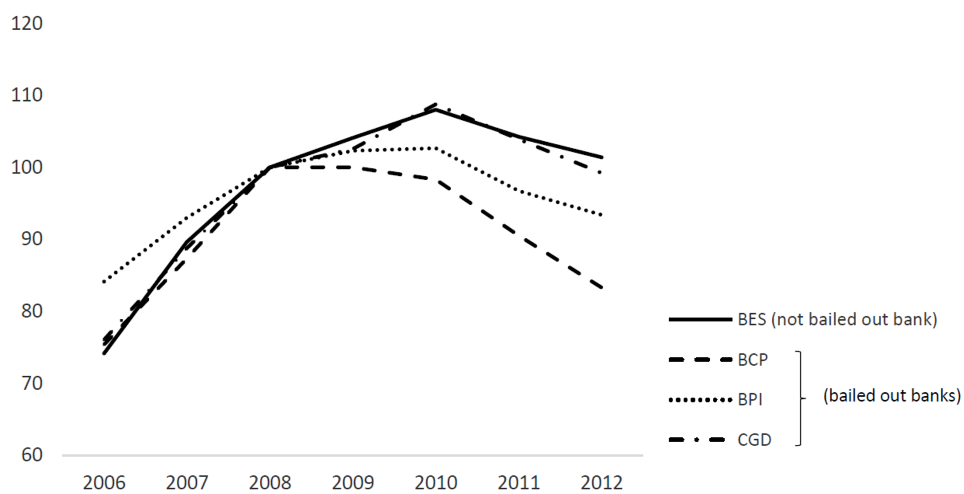

Figure 6: Effect of the 2009 credit shock on the lending behaviour by the four largest Portuguese banks. This figure plots total loans volumes considering 2008 as the reference year. We use Bankscope data for the four main Portuguese banks: Caixa Geral de Depóstios (CGD), Banco Comercial Português (BCP), Banco Português de Investimento (BPI) and Banco Espírito Santo (BES). Together these banks represent 60-70% of corporate debt.

6.4.2 Demand shocks

Another potential concern in our identification strategy is whether unobserved changes in investment opportunities may have biased our results. For example if the demand for goods produced by firms with strategic complementarities has fallen after 2009, then these firms would find optimal to reduce their production (and investment)—and such effect would not be related with the existence of spillover effects from strategic complementarities. Put more formally, our concern is that unobserved differences between both groups of firms trigger sharp contrasts in the post-crisis period because of changes in the environment other than spillover effects.

To address these concerns, we restrict our sample to exporting firms. The effect of the 2009 credit contraction on investment of exporting firms is very unlike to be explained by a reduction in the internal demand for their products, since these firms have the means to offset this reduction.

Column (3) of Table 4 shows the estimates of equation (1) when we restrict the sample to exporting firms which borrowed exclusively from non-bailed out banks in the period 2006-2012. The results are also statistical significant and with similar magnitudes to the baseline regressions. On average, firms with strategic complementarities reduce their investment (as a fraction of assets) by 3.12 percentage points more than firms without complementarities.

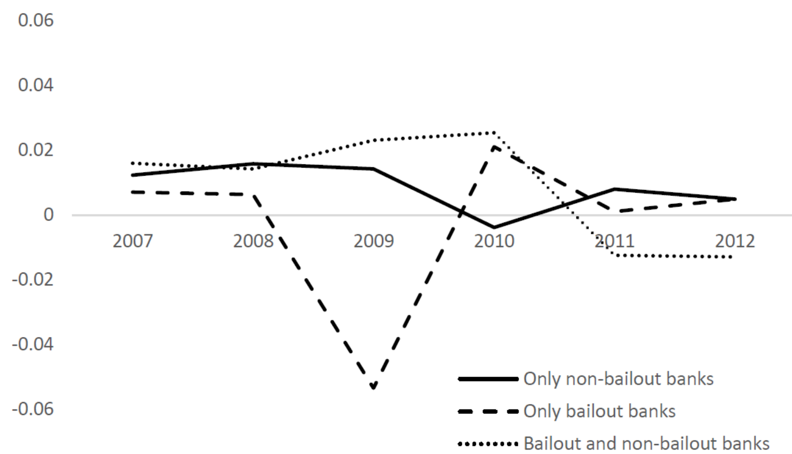

Figure 7: Evolution of firms' variation of debt over assets. This figure plots average values of the ratio year-on-year total debt difference over assets for (i) firms which during all the considered period of time worked only with non-bailout banks (solid line), firms which during all the considered period of time worked only with bailout banks (dashed line), and firms which work with both type of banks during the considered period of time (dotted line). Banks which were bailout: Banco Comercial Português, Banco Português de Investimento, Caixa Geral de Depóstios and Banco Internacional do Funchal. Banks which were not bailout: Banco Espírito Santo, Banco Santander, Banco Popular, Finibanco, Caixa de Crédito Agrícola, Banco Bilbao Viscaya, Barclays Bank, Montepio Geral, Fortis Bank, BNP Paribas, Caja de Ahorros, Deutsche Bank, Banco BIC, Banco Finantia, Banco Popular, ABN AMRO bank, Banco Privado, Banco Totta, Banco Best and Credit Lyonnais. We exclude the 10 firms which have borrowed from Banco Português dos Negócios, since this bank was nationalized in 2010.

Column (4) of Table 4 considers all exporting firms (regardless of whether they have borrowed from bailed out banks or not), and broadly confirms the previous results.

7 Counterfactual Matching Approach

Our main goal is to gauge how strategic complementarities affected firms in the aftermath of the 2009 credit shock. To this purpose we isolate the firms which benefit from strategic complementarities. We would like to compare their observed investment after 2009 (which was affected by spillover effects) with their *non-observed* investment *had their neighbors not been caught by the credit contraction*. Naturally this is a difficult task. One way to tackle this problem is to estimate the difference between the investment actually observed in the data and a plausible counterfactual investment. Since firms without strategic complementarities are not affected by spillover effects from their neighbors, these firms provide a natural counterfactual.

We conduct our analysis combining a difference-in differences approach with the use of a matching estimator. The idea behind this approach is that of isolating firms with strategic complementarities, and then, from the population of firms without complementarities look for *control* observations that best *match* the observations on firms with complementarities. We are assuming that *if it were not for the existence of strategic complementarities*, both groups of firms would have behaved similarly. The matches are made so as to ensure that observations in both groups have identical distributions along some pre-specified dimensions.

We employ the propensity score matching estimator of the "average effect of the treatment on the treated" proposed by Leuven and Sianesi (2003), using observed characteristics (such as assets, sales, cash flow, number of employees, and being an exporting firm or not) as inputs in a probit regression where the dependent variable is the dummy variable SC_i which identifies firms with strategic complementarities.[6] For each firm with strategic complementarities, the procedure finds the firm without complementarities with the closest propensity score. Once the assignment has been done, we can measure the difference-in differences in investment between both groups.

Table 5 shows that, on average, firms with strategic complementarities reduce their investment (as a fraction of assets) by 3.26 percentage points more than firms without complementarities. The magnitude of this estimate is comparable with the magnitude of the

[6] We could have applied the Abadie and Imbens (2011) estimator, which minimizes the Mahalanobis distance between the vector of observed covariates across treated and non-treated firms to find control firms. This estimator produces exact matches on categorical variables, but the matches on continuous variables are not exact. Given the relatively limited size of our sample, exact matches are sometimes unavailable. One way to deal with the problem of dimension in this setting is to use propensity score matching.

most demanding estimate we obtained with the sample splits (that is, when the sample was restricted to exporting firms which worked exclusively with non-bailed out banks), with the advantage of having a number of observations which is substantially larger.

8 Conclusion

We study the impact of the 2009 Portuguese credit crunch on firms with strategic complementarities. More specifically, we study the role of spillover effects on firms' investment decisions.

We find that corporate investment declines significantly following the onset of the credit crunch, controlling for firm fixed effects. On average, annual investment as a fraction of assets declined by 3.71 percentage points in the aftermath of the credit shock, which compares with the unconditional mean of 5 percent.

Consistent with a causal effect, the decline is greatest for firms with intense strategic complementarities. In our baseline regression, we estimate that annual investment (as a fraction of assets) declines by 2.17 percentage points more for firms with strategic complementarities.

To address selection bias and endogeneity concerns we restrict our sample to exporting firms which established banking relationships exclusively with banks which did not restrict their credit. Our goal is to isolate spillover effects among those among firms which did not see their credit restricted and suffered no change in investment opportunities. The estimate of the differential impact increases to 3.12 percentage points and continues to be statistically significant.

In a final step, we conduct our analysis combining a difference-in differences approach with the use of matching estimators. We estimate that firms with strategic complementarities reduce their investment (as a fraction of assets) by 3.26 percentage points more than firms without strategic complementarities, following the onset of the credit shock. The magnitude of this estimate is comparable with the magnitude of the most demanding estimate we obtained with the sample splits (that is, when the sample was restricted to exporting firms which worked exclusively with non-bailed out banks), with the advantage of having a number of observations which is substantially larger.

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9 Tables

Summary statistics

Panel A reports summary statistics for the variables used in the analysis. The sample period is 2006 to 2012. “Employees” is the number of firm’s employees. “Capital/Assets” is the ratio between capital (fixed tangible assets plus depreciations) and assets (total assets). “Investment/Assets” is the ratio between investment (fixed tangible assets plus depreciations in period t minus fixed tangible assets in period t-1) and assets. “Debt/Assets” is the ratio between total debt (long and short term debt) and assets. “CashFlow/Assets” is the ratio between cash flow and assets. “Sales” is the value of total sales. “Sales Variation” is the net sales growth rate. “Bank lending relationships” is the number of banks with which firms establish relationship. Panel B reports mean values for the same variables, distinguishing between firms with ("WithSC") and without ("WithoutSC") strategic complementarities. Differences in means are assessed with the t-test.

Panel A: mean, standard deviation, minimum and maximum for all observations

	Mean	StandDev	Min	Max
Employees (n)	48.75	44.41	10.00	246.00
Capital/Assets	0.30	0.21	0.00	1.23
Investment/Assets	0.05	0.24	-16.81	0.87
Debt/Assets	0.22	0.22	0.00	6.11
CashFlow/Assets	0.07	0.11	-2.69	1.01
Sales (euros)	4418.31	9912.47	0.00	243291.49
Sales Variation	8.60	35.23	-97.40	991.84
Assets (euros)	4612.42	13158.94	5.00	279324.54
Bank lending relationships (n)	3	2	1	11
N obs	7644			

Panel B: mean values for firms with and without strategic complementarities

	WithoutSC	WithSC	Difference	p-value
Employees (n)	43.21	50.09	-6.89	0.00
Capital/Assets	0.28	0.3	-0.02	0.00
Investment/Assets	0.06	0.05	0	0.54
Debt/Assets	0.23	0.21	0.02	0.00
CashFlow/Assets	0.06	0.07	-0.01	0.00
Sales (euros)	8,071.69	3,533.76	4,537.92	0.00
Sales Variation	8.09	8.73	-0.64	0.53
Assets (euros)	8,732.26	3,614.93	5,117.33	0.00
Bank lending relationships (n)	2.71	2.52	0.19	0.00
N obs	1490	6154		

Table 1: Summary statistics

The effect of the shock on firms' investment

The table presents results for the two groups of firms (with and without strategic complementarities) in which we compare investment before the onset of the crisis to investment after. In the comparison, we average each firm's time series into two sample means—one for the period 2006-2008, which we label as "before the crisis", and one for the period 2009-2012, which we label as "after the crisis". Differences in means are assessed with the t-test.

	Before the crisis	After the crisis	Difference	(p-value)
Without SC	0.066	0.050	0.016	0.005
With SC	0.077	0.037	0.040	0.000
N obs	2915	4729		

Table 2: The effect of the shock on firms' investment

The effect of the shock on firms' investment (regressions estimates)

This table shows estimates from panel regressions of the effect of a credit contraction shock on firms' investment, comparing firms with and without strategic complementarities. The dependent variable is firm's investment. Observations are at the firm-year level. Coefficients in columns (3) and (5) are estimated by random effects. Columns (1), (2), (4), (6) and (7) consider firms fixed effects. Control variables include sales (net sales variation), cash flows, debt and an exporter activity dummy. The variables' definition is provided in Appendix. Standard errors are clustered at the firm level. *, **, *** denote statistical significance at the 10%, 5%, and 1% levels, respectively and t statistics are in parentheses. SC - strategic complementarities dummy, CC - credit shock dummy, Sales - net sales variation, A - assets, Exporter - exporter dummy, RE - random effects, FE - fixed effects.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
SC			0.0091*		0.0092*		
			(1.69)		(1.68)		
CC	-0.0371***	-0.0204***	-0.0126**	-0.0201***	-0.0115**		
	(-10.67)	(-3.85)	(-2.26)	(-3.66)	(-2.05)		
CC*SC		-0.0209***	-0.0249***	-0.0212***	-0.0261***	-0.0217***	
		(-3.11)	(-3.34)	(-3.17)	(-3.39)	(-3.24)	
Sales			0.00019***	0.0001**	0.0002***	0.0001*	0.0001*
			(4.77)	(2.02)	(4.68)	(1.81)	(1.80)
CashFlows/A			0.156***	0.0708**	0.137***	0.0681**	0.0670**
			(4.79)	(2.33)	(4.87)	(2.04)	(2.03)
Debt/A			0.0420**	0.0426		0.0438	0.0445
			(2.21)	(1.50)		(1.50)	(1.50)
Debt/A_06					0.0006		
					(0.05)		
Exporter			0.0017	0.0212	0.0028	0.0208	0.0204
			(0.19)	(1.29)	(0.31)	(1.31)	(1.29)
DSC_D06							-0.009
							(-0.58)
DSC_D07							-0.0009
							(-0.10)
DSC_D09							-0.0445**
							(-2.20)
DSC_D10							-0.0278**
							(-2.15)
DSC_D11							-0.0145
							(-1.63)
DSC_D12							-0.0102
							(-1.02)
Constant	0.0758***	0.0758***	0.0437***	0.0462***	0.0531***	0.0336**	0.0339**
	(35.21)	(35.26)	(3.51)	(2.90)	(4.94)	(2.31)	(2.34)
RE			Yes		Yes		
Firm FE	Yes	Yes		Yes		Yes	Yes
Year FE						Yes	Yes
R^2	0.0069	0.0072	0.0079	0.0092	0.0077	0.0122	0.0126
N obs	7644	7644	7644	7644	7644	7644	7644

Table 3: The effect of the shock on firms' investment (regressions estimates)

Robustness checks

In column (1) we use the difference between the values of year t and of year $t-1$ for the total debt of the industry (normalized by assets) as the explanatory variable to determine the shock. Columns (2) – (4) shows estimates for appropriately selected subsamples. In column (2) we restrict the sample to firms which worked only with non-bailout banks. In column (3) we restrict the sample to firms which worked only with non-bailout banks and are exporters. In column (4) we restrict the sample to all exporting firms. Observations are at the firm-year level. All columns consider firms fixed effects. Control variables include sales (net sales variation), cash flows, total debt and an exporter activity dummy. The variables' definition is provided in Appendix. Standard errors are clustered at the firm level. *, **, *** denote statistical significance at the 10%, 5%, and 1% levels, respectively and t statistics are in parentheses. SC - strategic complementarities dummy, CC - credit shock dummy.

	(1)	(2)	(3)	(4)
DifDebt/Assets industry level (Debt)	0.0244			
	(1.07)			
Debt*SC	0.172**			
	(2.10)			
CC*SC		-0.0608***	-0.0312*	-0.0147**
		(-3.58)	(-1.90)	(-2.10)
Net Sales Variation	0.000142**	0.00000516	0.000262**	0.0000571
	(2.08)	(0.03)	(2.17)	(1.25)
Cash Flows	0.0914***	-0.0244	-0.0760	0.00737
	(2.64)	(-0.32)	(-0.94)	(0.32)
Debt	0.0302	-0.101**	-0.00223	0.0198
	(1.08)	(-2.14)	(-0.06)	(1.43)
Exporter dummy	0.0143	-0.00990		
	(0.92)	(-0.74)		
Constant	0.0262	0.0767***	0.0530***	0.0550***
	(1.58)	(4.85)	(5.81)	(12.02)
Firm fixed effects	Yes	Yes	Yes	Yes
Year fixed effects		Yes	Yes	Yes
R2	0.0031	0.2167	0.1030	0.0392
N obs	7644	574	367	5197

Table 4: Robustness checks

Counterfactual Matching Approach

This table shows the difference-in differences of firm investment before and after the credit crisis with difference-in differences estimator (DiD) and DiD matching estimator. For the DID we consider the most demanding sample-split with exporting firms which worked exclusively with non-bailed out banks, without control variables but controlling for firms fixed effects and standard errors clustered at the firm level. For the DiD matching estimator we employ the propensity score estimator of the "average effect of the treatment on the treated" proposed by Leuven and Sianesi (2003). *, **, *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

	Before the shock	After the shock	Difference	N obs
Without SC	0.0595	0.0631	0.0036	69
With SC	0.0595	0.0302	-0.0293	298
DiD	0	-0.0329	-0.0329*	367
DiD matching estimator (ATT)			-0.0326**	1103

Table 5: Counterfactual Matching Approach

A Appendix

List of industries proposed by Guimarães et al. (2007)

With Strategic Complementarities	
Industry Code	Industry Description
171	Preparation and spinning of cotton-type fibres
172	Cotton-type weaving
173	Bleaching and dyeing
176	Manufacture of knitted and crocheted fabrics
183	Tanning and dressing of fur
192	Manufacture of luggage, handbags and the like, saddlery and harness
193	Manufacture of footwear
223	Reproduction of sound recording
244	Manufacture of basic pharmaceutical products or medicaments
247	Manufacture of man-made fibres
263	Manufacture of ceramic tiles
296	Manufacture of hunting, sporting or protective firearms and ammunition
323	Manufacture of television and radio receivers, sound or video recording
332	Manufacture of instruments for measuring electricity, gas water and other fluid
334	Manufacture of optical non-ophthalmic instruments
341	Manufacture of motor vehicles
354	Manufacture of motorcycles and bicycles
362	Manufacture of filigree
363	Manufacture of musical instruments

Table 6: Industries with strategic complementarities

Without Strategic Complementarities	
Industry Code	Industry Description
242	Manufacture of pesticides and other agro-chemical products
268	Production of abrasive products and manufacture of bituminous mixtures
271	Manufacture of basic iron and steel and of ferro-alloys
272	Manufacture of steel tubes
273	Cold rolling of narrow strip and cold forming or folding and wire drawing
274	Aluminium, lead, zinc, tin, copper and other non-ferrous metal production
283	Manufacture of steam generators, except central heating hot water boilers
294	Manufacture of portable hand held power tools, other metalworking machine tools
297	Manufacture of electric and non-electric domestic appliances
311	Manufacture of electric motors, generators and transformers
322	Manufacture of television and radio transmitters
333	Manufacture of industrial process control equipment
335	Manufacture of watches and clocks
353	Manufacture of aircraft and spacecraft
364	Manufacture of sports goods
365	Manufacture of games and toys

Table 7: Industries without strategic complementarities

List applied exclusions on firms selection

	N firms	N obs (panel)
Firms in selected industries	8852	
Firms with >=10 and <250 employees	1945	
Firms with balance sheet information	5304	
… with information on Assets, Investment and Sales	1901	13401
… between 2006-2012	1392	7875
… excluding industries which depend on natural resources	1224	7644

Table 8: List of applied exclusions

Industries which depend on natural resources are: industry 232 - petroleum refining, industry 351 - shipbuilding and repairing, industry 152 - sea products processing, industry 160 - tobacco and industry 372 - recycling of non-metallic products.

Variables definition and Sabi codes

Variable Name	Description	Sabi Codes
id	Firm fiscal number	
CAE	Industry (CAE Rev. 2.1)	
d_withsc	Firms with SC identification	
t	Balance sheet year	
Number_Employees	Number of employees	747
D0i	Time dummy for 200i, i:6-13	
DC0i	Interaction term (d_withsc x D0i)	
Assets	Assets	706
TFAssets	Tangible fixed assets	734
TFAssets_Assets	Tangible fixed assets over Assets	734/706
Capital	Capital (TFAssets+Depreciation)	734+745
Capital_Assets	Capital assets over Assets	(734+745)/706
Investment (Capex)	(TFAssets+Depreciation)$_t$ - (TFAssets)$_{t-1}$	(734+745)$_t$ - (734)$_{t-1}$
Investment_Assets	Investment over Assets	((734+745)$_t$ - (734)$_{t-1}$)/706
Debt_Assets	Debt over Assets	(738+729)/706
VarDebt	Debt$_t$ - Debt$_{t-1}$	(738+729)$_t$ - (738+729)$_{t-1}$
VarDebt_Assets	VarDebt over Assets	((738+729)$_t$ - (738+729)$_{t-1}$)/706t
CashFlow	Cash Flows	717
CashFlow_Assets	Cash Flows over Assets	717/706
Sales_DemResult	Sales	727
NetSalesVariation	Net sales growth rate	86

Table 9: Variables definition and Sabi codes